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Metallic Compression Members

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METALLIC COMPRESSION MEMBERS

BY

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I hereby recommend that the thesis prepared under my supervision by RAYMOND JEFFERSON ROARK entitled METAL COMPRESSION MEMBERS be accepted as fulfilling this part of the requirement for the Degree of Master of Science.

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I. INTRODUCTION.

Of the many problems which have engaged the attention of the structural engineer, few have been at once so important and so difficult of satisfactory solution as that presented by the analysis and design of columns. Theory has failed because of the large number of modifying conditions which cannot well be taken into account. Most experiments have been unsatisfactory because of the fact that they show only the ultimate strength of the specimen, and that under conditions of loading neither uniform nor similar to those occurring in actual structures.

From time to time failures of structures have served to emphasize the uncertainty of our knowledge concerning columns, and the importance of acquiring more definite information upon this subject. The greatest impetus to the study and investigation of metallic columns was probably given by the failure of the Quebec bridge. This failure showed, particularly, the importance of securing a proper form of section, and correct proportion of auxiliary parts. It became evident that the inherent properties of the material and the ratio of length to radius of gyration were not the only factors controlling the strength of columns, and that the rules of design which had been considered generally applicable might well be at fault when carried beyond a certain point.

The various tests which have been made subsequent to the Quebec Bridge failure have shed light on several disputed points, such as the stresses which may be expected to exist in the parts of a built up column, the effect of shape of section and general make up, and other questions difficult or impossible of theoretical

treatment.

It is purposed in this paper to give an outlino of knowledge concerning metallic columns and to present several new ideas. A general idea of the discussion is given by the table of contents. Most of the matter of Part II is taken from various works on mechanics and from papers printed in the transactions of the American Society of Civil Engineers. Part III consists partly of a discussion of various experiments on columns. An original analysis of the strength of outstanding flanges is attempted, and the results obtained compared with the results of tests. Part IV presents the results of tests made to determine something of the effect of riveting on the strength and elastic properties of compression members. Part V consists of a summary of the conclusions regarding the various questions discussed.

II. COLUMN FORMULAS.

Art. 1 Introductory.

The problem of determining, by analysis, the strength of columns and the stresses which exist in such members has never been satisfactorily solved. Many formulas have been developed, the majority of which are intended to define the breaking strength of a column of given dimensions. Most of these formulas are more or less empirical, and all depend upon certain assumptions the validity of which is open to question.

In the following discussion of column formulas, distinction will be made between those which apply to members under nominally axial loading and those which apply to members under definitely eccentric loading. The following formulas will be considered with regard to their significance, the theory and assumptions incident to their derivation, their agreement with experimental results, and their practical application:

For axial loading:

Euler's formula

Rankine's formula

The straight line formula

The parabolic formula

For eccentric loading:

The "Moncreff-Merriman" formula

Navier's formula.

The nomenclature used is as follows:

p = unit load in pounds per square inch.

E = modulus of elasticity, taken as 30 000 000 pounds per square inch for steel.

l = length of column in inches.

r = least radius of gyration of the column section in inches

f = extreme fiber stress in pounds per square inch.

Art. 2. Axial Loading.

The first successful attempt to derive a mathematical expression for the strength of columns was made by Euler in 1757. He developed the formula

$$p = \frac{c\,\pi^2 E}{\left(\frac{l}{r}\right)^2}$$

This formula indicates the maximum load that a given column will support without buckling. Its derivation is based on the equation of the elastic curve and may be considered theoretically correct. The value of the constant C depends upon the end conditions, and is usually taken as 1 for round ends, 2.05 for one end fixed and one end round, 4 for both ends fixed, and 1/2 for one end fixed and the other free.

As stated, this formula applies rigidly to failure by buckling, but it takes no account of the compressive strength of the material. For ratios of $\frac{l}{r}$ sufficiently great to insure failure by bending before the elastic limit of the material is passed, the values for the critical load as given by Euler's equation conform very closely with the results of experiments. For small ratios of $\frac{l}{r}$ the values given are much too high. In any event the formula may be regarded as defining, with practical accuracy, the upper limit of column strength.

It is apparent from this formula that no column can be expected to permanently carry a load which causes an average stress above the yield point of the metal. When the yield point is reached

the modulus of elasticity, or more accurately, the ratio of increment of load to increment of deformation, becomes very small and the column fails by buckling approximately as represented by Euler's formula with a greatly reduced modulus of elasticity.

Owing to the fact that it gives the actual strength only for those columns having extremely high slenderness ratios Euler's formula is not ordinarily applicable in column design or investigation.

With the object of developing a formula which would take into account the compressive strength of the material, Rankine, in 1860, derived the following expression:

$$f = p\left(1 + \phi \frac{l^2}{r^2}\right)$$

This formula is designed to give the maximum stress existing in a column of given dimensions under a given load. Its derivation is based on the assumption that the deflection of a column, like that of a beam, varies directly as $\frac{l^2}{c}$ for a given fiber stress The value of the constant ϕ is determined from experiment and the generally accepted values are $\frac{1}{25,000}$, $\frac{195}{25,000}$ and $\frac{4}{25,000}$ for steel columns with both ends fixed, one end fixed and one round, and both ends round respectively. No account is taken of initial deflection. The stress due to direct compression and that due to the bending are added directly.

It can be seen that this formula is derived by analogy instead of by rigid theory, and is, therefore, less rational than Euler's. Furthermore, the value of the constant ϕ is determined by tests carried to failure and hence the formula is not only largely empirical but is based on results obtained under conditions quite different from those existing under working loads. It does not therefore appear reasonable to suppose that the stresses calculated

by this formula will agree with those actually existing in a column. This conclusion is supported by recent experiments in which careful strain measurements at different points failed to indicate any such distribution or intensity of stresses as would be inferred from the formula.

When ϕ is carefully determined by experiment for a given form of column Rankine's formula may be relied upon to give values for the ultimate strength very close to the real values. When the value of ϕ is taken as the same for all forms of columns of the same material, the results are less satisfactory. The formula is convenient and has been used in engineering practice perhaps more than any other. Its application to cases where the stresses in the parts of the column under working conditions are to be found does not, however, appear to be warranted.

A study of column experiments shows that in many cases the results may be represented by a straight line quite as well as by the curve of Rankine's formula, within certain limits of the ratio $\frac{l}{r}$. On the basis of this fact, T. H. Johnson deduced the well-known formula:

$$p = f - c\frac{l}{r}$$

This formula is, of course, largely empirical. The equation of the line is determined by making f the ultimate compressive strength of the material and drawing the line tangent to the curve of Euler's formula. Johnson gives as values of c, 220 for hinged ends and 179 for flat ends. He takes f = 52,500 for mild steel. The straight line formula is very convenient for use in practice and has been widely adopted in specifications. When used for design the entire equation is divided by a suitable factor of safety.

With the object of obtaining an expression somewhat rational and at the same time convenient of application, the following formula was developed.

$$p = f - c\left(\frac{l}{r}\right)^2$$

This formula, which was proposed by Professor Johnson, is similar to the straight line formula in that its derivation is largely empirical. The equation represents a parabola drawn tangent to Euler's curve and having its vertix at the elastic limit on the load axis. Professor Johnson gave for steel columns $f = 42,000$ and $c = 0.97$ and 0.62 for pin-ends and flat-ends respectively. He uses as the corresponding equations for Euler's formula

$$p = \frac{456,000,000}{\left(\frac{l}{r}\right)^2}$$

for pin ended columns and

$$p = \frac{712,000,000}{\left(\frac{l}{r}\right)^2}$$

for flat ended columns. Using these figures the parabola becomes tangent to Euler's curve at $\frac{l}{r} = 150$ and $\frac{l}{r} = 190$ for the two cases.

It is claimed that this formula is easy of application and that it agrees very closely with the results of experiments.

Discussion of Formulas. In order that the above formulas may be compared the curves representing the several equations have been plotted, see Plate I. Euler's curve is drawn using the values recommended by Johnson for pin-ended columns, these values being intermediate between those for round-ended and those for flat-ended columns. Rankine's formula

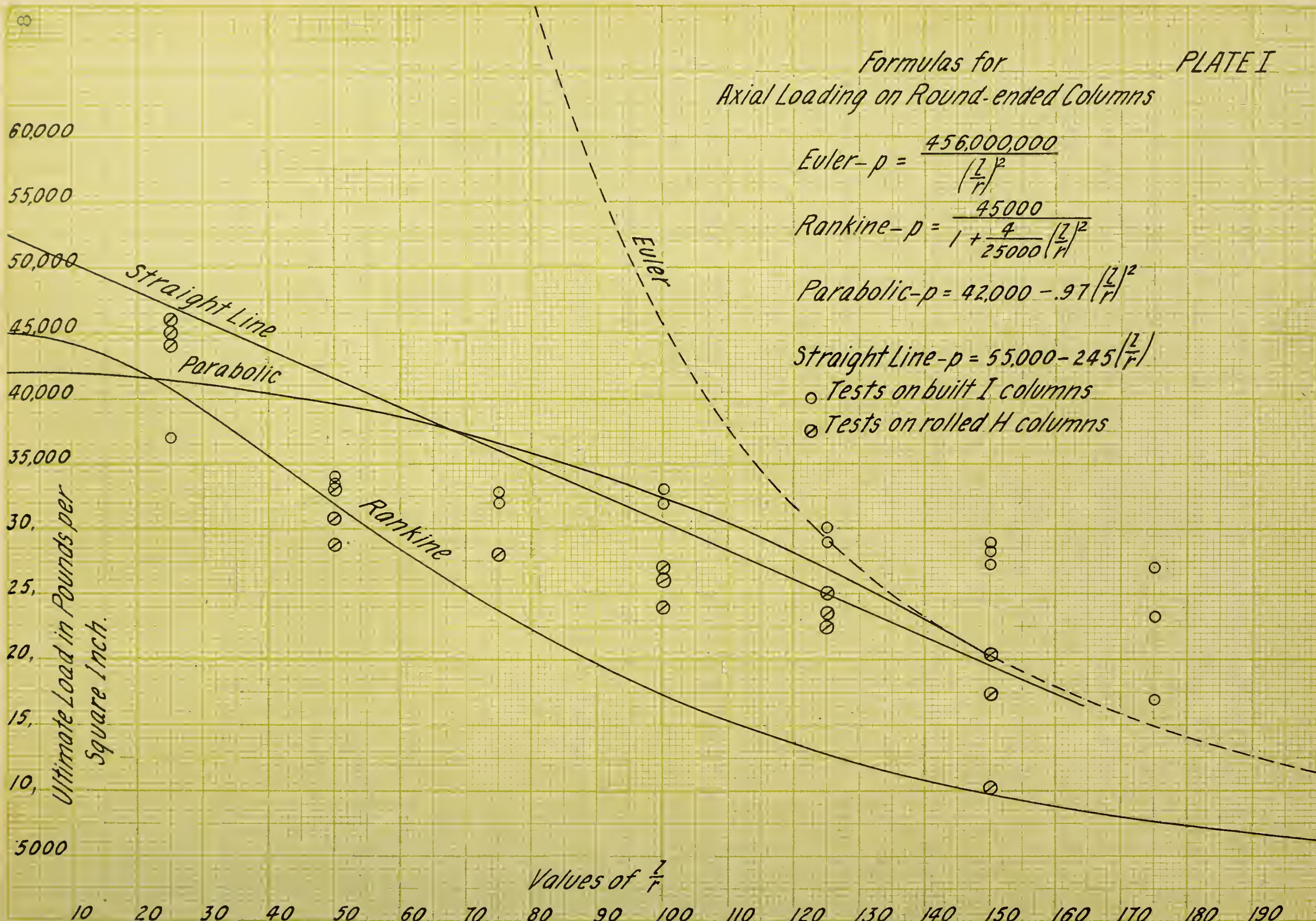
PLATE I
Formulas for
Axial Loading on Round-ended Columns
Euler – p = 456,000,000 / (l/r)²
Rankine – p = 45000 / (1 + 4/25000 (l/r)²)
Parabolic – p = 42,000 – .97 (l/r)²
Straight Line – p = 55,000 – 245 (l/r)
Tests on built I columns
Tests on rolled H columns
Ultimate Load in Pounds per Square Inch.
Values of l/r
Euler
Straight Line
Parabolic
Rankine
60,000
55,000
50,000
45,000
40,000
35,000
30,
25,
20,
15,
10,
5000
10
20
30
40
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60
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has been plotted using for "f" the average value for ultimate strength obtained in the Watertown arsenal tests on very short columns. The constant used is that given by Merriman for round ends. The parabolic formula is drawn as given by Professor Johnson for pin-ended columns and the straight line formula as given by T. H. Johnson for round-ended columns.

In order to enable a comparison to be made between the values given by these formulas and the results of experiment, the results of tests made at Watertown arsenal on pin-ended columns have been plotted on the same sheet with the curves. These tests were carried to destruction, the values obtained for ultimate strength being plotted. Pin-ended columns only are considered as it is believed that experiments upon this type are more reliable and representative. As a matter of fact the results obtained on flat-ended columns differed but little from those for pin-ended columns up to a value of $\frac{l}{r}$ of 150. The tests were made upon lap-welded steel tubes, rolled H-sections, and built-up I-sections.

It can be seen from the curves and test values here plotted that none of the formulas can be depended upon to give accurate results for one type of column when used with constants derived from tests made on another type. The difference between the strength of columns of equal slenderness-ratio but different form is seen to be as great as that between columns of similar form and very different slenderness-ratio. Furthermore, to judge from the experiments referred to the effect of slenderness may be greater in the case of one type of column than another. Thus the rolled H-columns show a much more rapid decrease of strength with increase of $\frac{l}{r}$ than the built-up I-columns. The fact that considerable initial deflection was present in some cases, while in others the columns were practic-

ally straight, undoubtedly accounts for part of the variation in results. Thus, in the case of the three longest built I-columns, $\frac{l}{r}$ = 175, considerable difference may be noted in the results. The column having the greatest strength was, as nearly as could be determined, perfectly straight. The next strongest column had an initial deflection of 0.15 inches horizontally, and the weakest column had an initial deflection of 0.14 inches vertically and 0.15 inches horizontally. The three longest rolled H-columns, $\frac{l}{r}$ = 150, also gave widely varying results. The strongest had an initial deflection of 0.15 inches horizontally and 0 inches vertically, the next strongest of 0.16 inches horizontally and 0.22 inches vertically and the weakest, which failed under a unit load of 10,100 pounds per square inch had an initial horizontal deflection of 0.42 inches. These deflections are greater than was the case with the built-up I-columns; and this fact undoubtedly accounts in great measure for the low strength obtained.

These and other tests appear to indicate that initial deflection or eccentricity, even in the case of nominally axial loading, may so greatly affect the strength of a column as to render the effect of variation in slenderness-ratio practically negligible between certain limits. When results are taken for values of $\frac{l}{r}$ ranging all the way from 0 to 200 the variation in strength is, of course, marked. If values obtained for columns having ratios of $\frac{l}{r}$ between say 50 and 100 be noted, however, it will be seen that but little variation can be definitely ascribed to the effect of slenderness-ratio between these limits.

There is another important point to be noted in this connection. Nearly all of the tests of columns which have been relied upon in determining the constants for formulas have been made by constant-

ly increasing the load until failure occurred. Higher values are obtained for the ultimate strength in this way than is the case when the column is subjected to a constant load for a considerable period. It has been pointed out that the elastic limit, or at most the yield point, is the ultimate strength of a column according to Euler's formula. In short columns, however, the failure at yield point is slow and will occur only under a continued load. If a further load is applied with comparative rapidity the column will develop a higher strength; but such strength could not be considered available in structures subject to constant load. The above conclusions are borne out by experiments in which time loading was employed. For unit loads above the elastic limit a steadily increasing deflection was observed with no increase in load.

It may thus be seen that for all members short enough to resist failure by elastic deflection and long enough to be regarded as subject to column action, the elastic limit may be regarded as the practical ultimate strength. When, moreover, the importance of secondary flexure, shear and other stresses is considered, the use of the ordinary formulas for such cases as commonly occur in practice does not seem to be fully warranted. It may be remarked in this connection that the shortest columns in the Watertown tests, having a ratio of $\frac{l}{r}$ = 25, failed by secondary flexure in the flanges. It is possible, by a proper choice of scale and of constants, to show a fair agreement between the results of any one series of tests and almost any of the column formulas. The agreement is, however, apparent only when a wide range of $\frac{l}{r}$ is considered. For values of $\frac{l}{r}$ such as ordinarily occur in practice the agreement cannot be regarded as close and often a horizontal line will be found to fit the results of tests as closely as the curves of the formulas. If the values

plotted on page be regarded indiscriminately it will be seen that from $\frac{l}{r} = 50$ to $\frac{l}{r} = 150$ a horizontal straight line could be made to express the results quite as well as any one of the curves.

In consideration of the above facts, it would seem permissable to adopt, for columns having slenderness-ratios between certain limits, constant working unit loads based upon the yield point of the material. A higher factor of safety than is used for tension should, of course, be provided. In determining such an allowable unit load the form of the member and its position in the structure should be considered as well as the properties of the material.

The adoption of a fixed working load for columns of usual dimensions would not result in a very marked departure from common practice. Practically all of the important compression members used in ordinary structures have ratios of $\frac{l}{r}$ between 30 and 100. Usually the ratios lie between much narrower limits. Suppose a constant working load be adopted between the limits $\frac{l}{r} = 40$ and $\frac{l}{r} = 80$. Between these limits Cooper's specifications give a range in allowable unit load of from 22 to 26 per cent, depending upon the member. The specifications of the American Railway Engineering Association give a range of about 21 per cent. If a constant working load were used between these values of $\frac{l}{r}$, equal to the mean of the extreme values given by the specifications, the maximum departure from practice would be about 12 per cent. This appears to be a considerable difference, but as a matter of fact, tests made in the same manner on similar columns frequently give results varying much more widely than 12 per cent.

Art. 3. Eccentric Loading.

In a paper entitled "The Practical Column" read before the American Society of Civil Engineers in 1901 by Moncrieff the following formula was proposed as applicable to columns under eccentric loading.

$$f = p\left[1 + \frac{ce}{r^2}\left(\frac{p\,l^2}{8Er^2 - \frac{5}{6}pl^2} + 1\right)\right]$$

Moncrieff derived the above formula from the assumption that the elastic curve of the column was a parabola. He obtained as the expression for deflection $\Delta = \dfrac{pa\ l^2\ e}{8EI - \frac{5}{6}\,pa\ l^2}$ and combined the direct stress with the stress due to the bending moment $pa\ (\Delta + e)$.

Merriman derived, from the equation of the elastic curve, the following expression for the maximum stress in a column eccentrically loaded.

$$f = p\left(1 + \frac{ce}{r^2}\ \sec\theta\right) \quad \text{where} \quad \theta = \frac{pa\ l^2}{EI}$$

This formula is theoretically correct, but sec θ can of course be only approximately evaluated. If sec θ be taken equal to $\dfrac{12 + \theta^2}{12 - 5\theta^2}$, which is very nearly correct, the above formula becomes identical with that of Moncrieff.

In this formula the effect of end conditions is taken into account, as in Euler's equation, by varying the effective length.

Another expression for the stress in an eccentrically loaded column is:

$$f = p + \frac{paey}{\left(I - \frac{pal^2}{CE}\right)}$$

This equation is derived by considering the deflection of a column in terms of extreme fiber stress, to be given by the expression $= \frac{fl^2}{cEy}$. This assumes conditions in a column to be analogous to those in a beam. The moments due to initial eccentricity and to deflection are added directly. C is taken as 10 for round-ended columns, 24 for one end hinged and one fixed, and 32 for both ends fixed. It can be seen that this formula is less rational than the one first discussed, but it has met with general acceptance.

<u>Comparison of formulas</u>. Experiments upon eccentrically loaded columns are too few to enable a comparison being made between the values given by the above formulas and the results of tests. It will be of interest however, to compare the two formulas one with the other.

Both equations can be written in the form

$$f = p \left(1 + \phi \frac{My}{I}\right)$$

In the Moncrieff formula

$$= \frac{48E + p\left(\frac{l}{r}\right)^2}{48E - 5\,p\left(\frac{l}{r}\right)^2}\text{, in the Navier}$$

formula

$$= \frac{1}{1 + \frac{p}{10E} \cdot \left(\frac{l}{r}\right)^2}\text{.}$$

For both cases the term $p\left(\frac{l}{r}\right)^2$ may be regarded as the independent variable. The curves, Plate II, have been plotted with values of $p\left(\frac{l}{r}\right)^2$ as abscissae and values of ϕ as ordinates. The curves are plotted up to $p\left(\frac{l}{r}\right)^2 = 140{,}000{,}000$, corresponding to a value of $p = 14{,}000$ and $\frac{l}{r} = 100$.

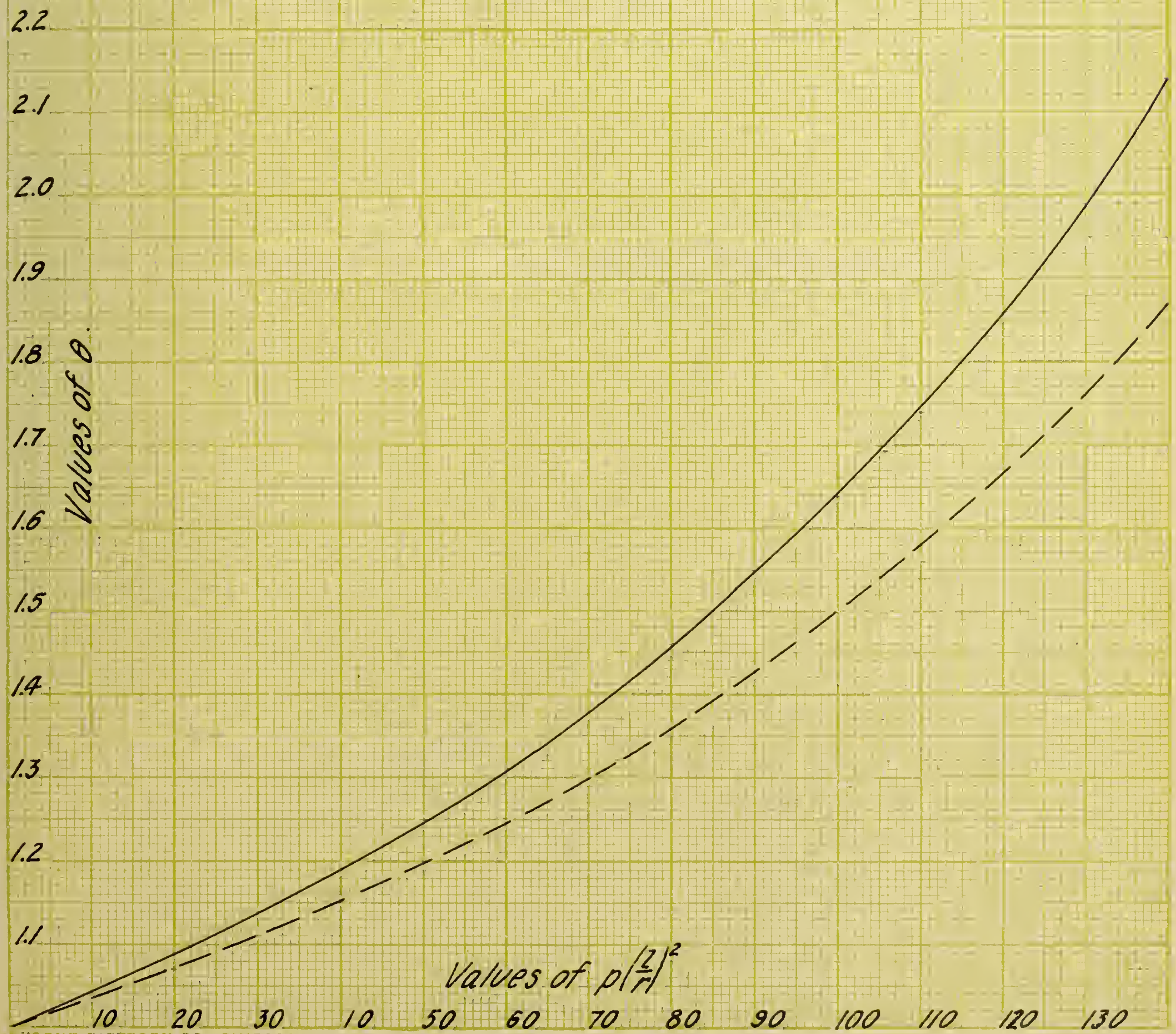
PLATE II

Formulas for
Eccentric Loading on Round-ended Columns

$$f = p + \theta \frac{Mc}{I}$$

—— Moncrieff-Merriman Formula, $\theta = \dfrac{48E + p\left(\frac{l}{r}\right)^2}{48E - 5p\left(\frac{l}{r}\right)^2}$

– – – Navier Formula, $\theta = \dfrac{1}{1 - \frac{p}{10E}\left(\frac{l}{r}\right)^2}$



It will be seen that the curves are of the same form, but that the Moncrieff formula gives higher values. By using a lower value of c the Navier formula could be made to nearly coincide with the Moncrieff formula.

III. STRESSES IN BUILT-UP COLUMNS.

Art. 4. Distribution of Compressive Stress.

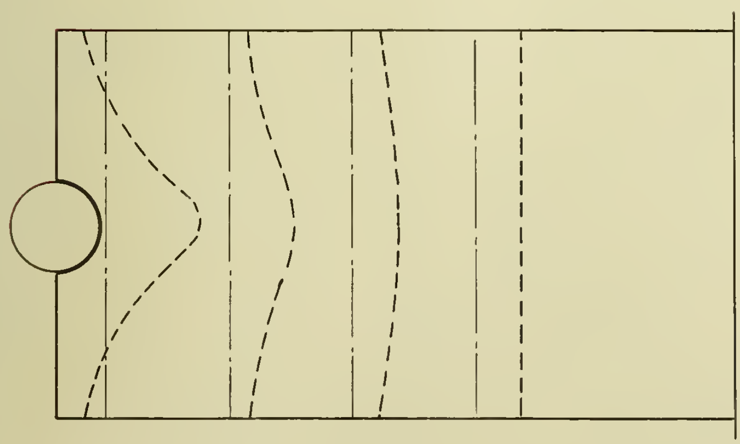
The theory underlying the column formulas discussed in the fore-going article assumes unity of action in the column parts and homogeneity of the member. The ordinary structural steel compression member, however, is not a unit, but is made up of several parts more or less rigidly held together. There is, therefore, some question as to just how nearly unity of action and integrity of section is preserved in such a member when subjected to load. The manner in which the stress is distributed among the component parts and the nature and intensity of local secondary stresses are important questions and have a direct bearing on the proper design of columns, especially those of large size.

Owing to the difficulty of investigation along this line comparatively little was known on the subject until recently. Many of the conditions governing cannot be taken into account in a theoretical treatment and the difficulty of making the numerous and accurate measurements necessary renders an experimental study tedious and expensive. Within the past few years, however, several series of tests have been made which serve both to throw light on the question and to further emphasize its importance.

In the present discussion of this subject a consideration of these tests will constitute the most important part; but an attempt will be made to treat certain topics from a theoretical standpoint as well. The distribution of compressive stress across the section and along the length of the member, the nature and magnitude of secondary stresses in the main parts, and the stresses in the lattice bars and other auxiliary parts, will be considered in order.

A column consisting of webs and angles held by lattice bars and loaded axially through pins will be assumed. One side only need be considered. Half the applied load is transmitted directly from the pin into the metal of the web and pin plates immediately ahead. The stress is transmitted along the axis by compression and across the section to the outer fibers by shear. The conditions are represented graphically in Fig. 1. Immediately in front of the pin is a zone of high stress. Further toward the edges of the column at the same section the compressive stress across various sections may be represented by the curves shown. It will be seen that as the center of the column is approached the stress rapidly becomes more evenly distributed across the section, being practically uniform at a very short distance from the ends. This distribution of the compressive stress can only be brought about by high shearing stresses and it is evident that the wider and thinner the plates the more intense and serious these shearing stresses become. Failure through true shear is improbable, but the greater the shearing-distortion the greater the local intensity of compressive stress.

Fig. 1



When the parts are thin, this intense compressive stress may well result in failure through local buckling. It is thus apparent that in order to secure even distribution of stress across the section and to avoid excessive local stress the sides of the column should be very thick and rigid for some distance in front of the pins. It is not improbable that the ordinary specification, limiting as it does the bearing pressure only, results in too little metal in front of the pins.

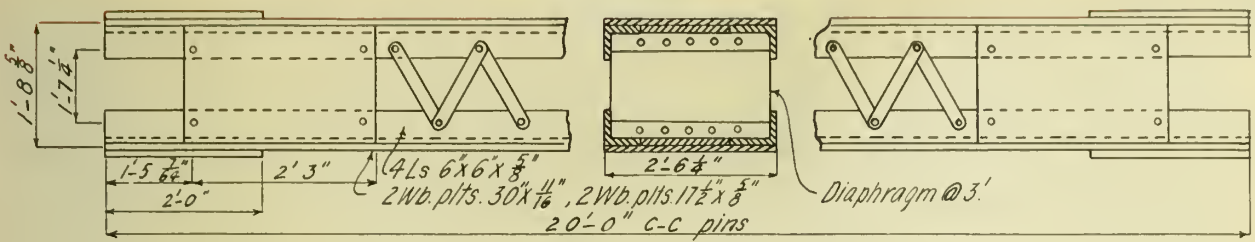
Tests made at the Watertown Arsenal by Howard, on columns similar to that assumed above, show the distribution of compressive stresses through the member. Measurements were taken over 10-in. gauge lengths at both ends of the column, but only on one side. The make-up of the columns and the strains observed are shown on Plate III. It will be seen that the results agree with what has been said in the above discussion.

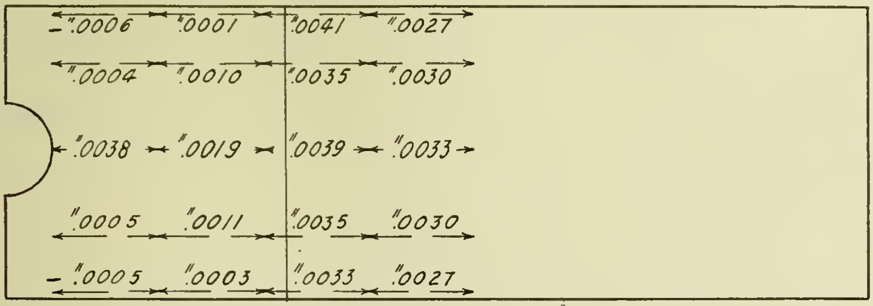
It might be possible, through the application of the theory of elasticity, to determine the actual stresses existing in the column adjacent to the pins. Such an exact analysis, however, is unnecessary and will not be attempted.

There are, in any built-up column, certain conditions which give rise to local secondary stresses. There are kinks and bends in the flange angles, plates and other parts due to fabrication and to accidents in handling. Under load the metal at such points will be stressed irregularly and local stresses of high intensity may be set up. A very small bend, especially if sharp, is sufficient to cause very high stresses. There are also initial strains in the metal at various points due to cold-straightening, cooling, punching and so forth. The result is that some fibers are stressed

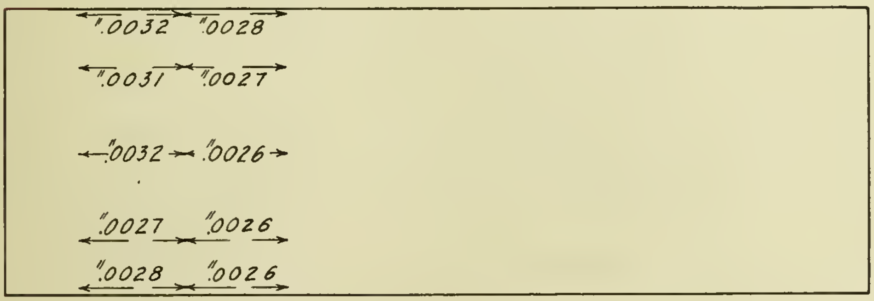
TESTS BY HOWARD

PLATE III

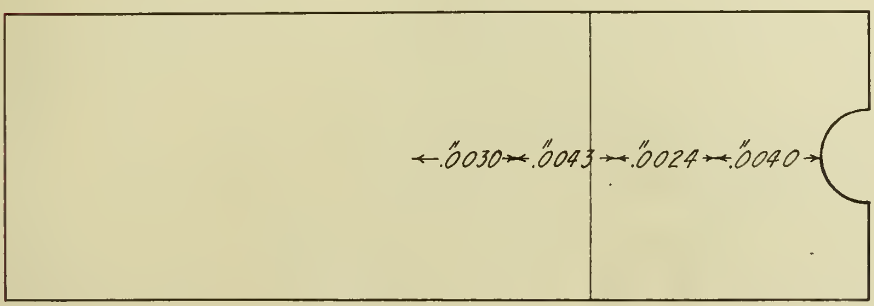




Compressive strains. North end. Measured on gauged lengths of 10 inches. Load increment 8597 pounds per square inch.



Corresponding strains at middle.



Corresponding strains at south end.

beyond the elastic limit and suffer a permanent set long before the unit load on the column has reached a maximum, or even a working value. The tests made by Howard, which have been referred to above, show the effect of such initial and secondary stresses on the elastic behavior of the column. Up to a unit load of about 15,000 pounds per square inch the stress-strain curve for the column as a whole was practically a straight line. Beyond that point the curve began to deflect, assuming a distinct, though not a sharp, curvature. This departure from the straight line at such a low unit load indicates that the metal at certain points had been overstrained and was not effectively resisting compression. Up to a unit load of about 9,000 pounds per square inch the modulus of elasticity, figured from the shortening of the entire column, was 29,375,000 pounds per square inch Howard concluded that this indicated substantial unity of action between the parts of the column. This is a reasonable supposition, but cannot be regarded as an assured fact. An important considera-tion is the effect that this irregular distribution of stress would be expected to have on the ultimate strength of the column. It is evident that when the metal at any point has been strained beyond the yield point the effect is to change the form and area of the effec-tive section. The center of gravity of the effective section moves away from the overstrained area, causing a certain eccentricity. The result is that the line of pressure throughout the length of the column is not straight, but irregular, and bending moments of greater or less intensity are indured at various points. If instead of the above mentioned condition, where only a small area of metal is weakened, the more serious condition of initial flexure in a flange angle or web obtains, the effect on the column is more marked. That

small bends, due to processes of fabrication, handling, and erection, do exist in the component parts of a column is certain. At such bends the extreme fibers are over-strained, and the part is in a very poor condition to carry load. The result is that the stress is concentrated in adjacent parts, or, if borne by the injured portion, causes a yielding of the fibers that results in the column as a whole tending to bend.

Tests made on built-up columns, where strain measurements were taken at various points, indicate that the stress distribution is very irregular. Careful tests made at the University of Illinois, Bulletin No. 44 of the Engineering Experiment Station, showed an excess of maximum stress to average stress of as much as fifty per cent. The distribution of stress across the section and along the length was found to be very irregular.

The significance of the foregoing discussion is, briefly, that the distribution of stress across the section of a built-up column is not uniform, but more or less irregular, and this distribution varies throughout the length of the member. The constituent parts are not necessarily evenly stressed, and do not act together perfectly. The result is that an eccentricity, varying at different sections, obtains even with a centrally applied load. So long as the unit load does not pass the yield point of the material a properly proportioned column will resist the bending moment indured by this eccentricity, but as soon as the yield point is passed the elastic resistance is destroyed and gradual buckling results.

Art. 5. Secondary Flexure.

In a column composed of relatively thin webs laced together at the edges or of members having wide outstanding flanges, failure may occur by local flexure or wrinkling of these parts before the full strength of the column as a whole has been developed. It is evident that there is a point where the reduction in thickness of metal in order to secure an increased section modulus with a given area results in a weakening of the member.

An exact analysis of the stresses in a flange or web of this kind would be extremely difficult, if not impossible. It would combine the difficulties of column and flat plate investigation, and in the present state of knowledge may be regarded as impracticable. By making certain assumptions, however, some relation between the strength and dimensions of such a member can be determined, which, while not accurate, may serve to show in a general way the effect of varying proportions. An attempt will be made to determine such a relation for the case of an outstanding flange.

Let: l = length of the member in inches;

b = width of flange;

t = thickness of flange;

p = unit load on the flange in pounds per square inch;

y = the lateral deflection of any point along the flange from its original positions; and

E = modulus of elasticity of the material in pounds per square inch.

In Fig. 2 the flange is shown. It is assumed to be rigidly held along the base A-B by the rest of the column.

Any elementary transverse strip of the flange, as a-b, may be regarded as a cantilever beam. Any elementary longitudinal strip, as c-d, may be regarded as a column. The load on each of these elementary columns causes a tendency to bend, and this bending is resisted partly by the column itself and partly by the reactions r of the elementary cantilevers against the column. Conversely, each cantilever is loaded by the lateral thrust of the elementary columns, and since the thrust of any column is proportional to its middle deflection, each cantilever is loaded at any point according to its deflection at that point. This gives a condition of loading intermediate between that of a uniform and that of a concentrated load. Since the error involved is slight, the elastic curve of each elementary cantilever beam will be assumed to be a parabola, the difference between the sinusoid given by Euler's theory and a parabola being slight for small curvatures.

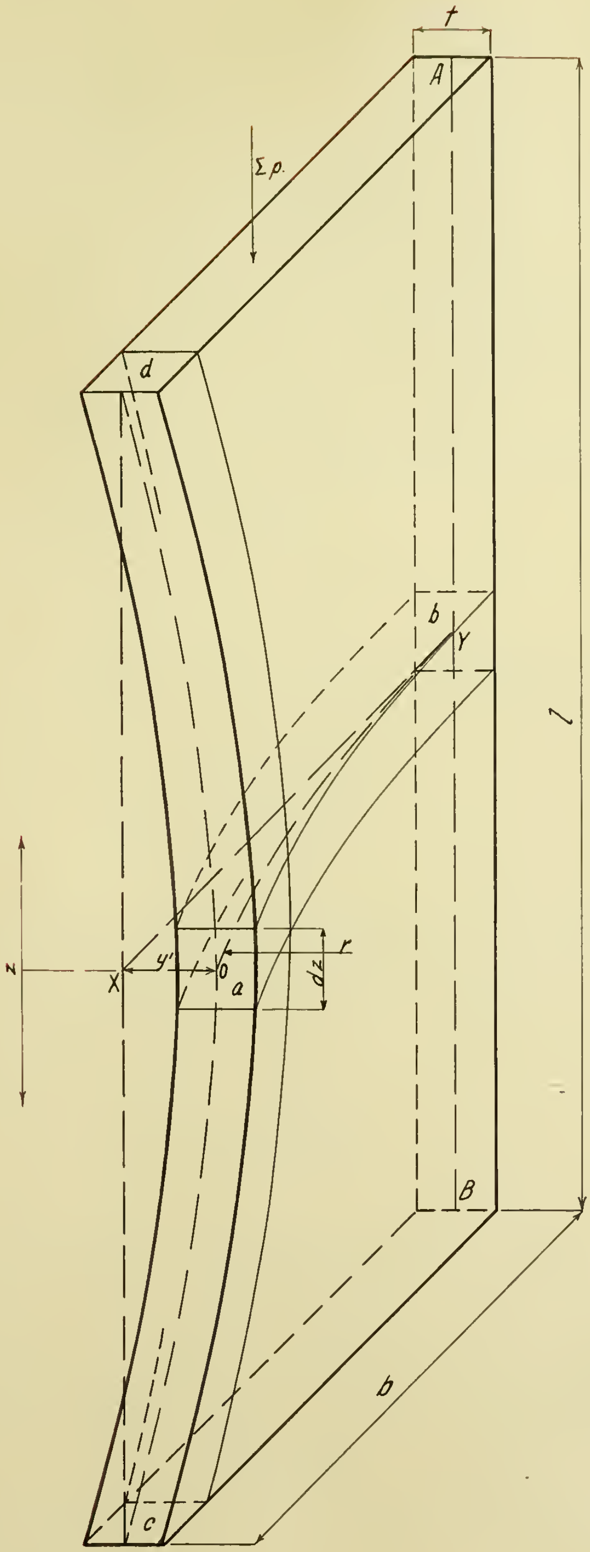
Since the elastic resistance of a long column is independent of its deflection, each of the elementary columns in the flange will carry the same load. Hence the entire flange may be taken as a single column and the load carried by it directly, as given by Euler's formula, is

$$\sum p = \frac{\pi^2 E\ t^3 b}{12\ l^2}$$

The remaining load, which will be denoted by P', is equal to

$$P' = ptb - \frac{\pi^2 E\ t^3 b}{12\ l^2} \qquad (1)$$

Fig. 2.



This remaining load is resisted by the cantilever action of the flange, and hence causes a certain bending moment along A-B. It is desired to find this bending moment.

Let any elementary column, as c-d, be considered. As in the case of the entire flange, a certain portion of the load coming on this elementary column is resisted by column action. The remainder of the load, which will be denoted by p', is resisted by the combined reactions r of the elementary cantilevers. If all these reactions were concentrated at o, the middle of the column, the amount required to balance p' could be found as follows.

Taking moments about o

$$2\,p'y = \sum r\,\frac{l}{2}\quad,\quad\text{hence}$$

$$\sum r = \frac{4\,p'y}{l}$$

Again, if the reactions acted uniformly along the length of c-d the amount required would, from analogy with the case of a simple beam, be just twice as great and we would have

$$\sum r = \frac{8\,p'y}{l}$$

Since the reactions are neither concentrated nor uniformly distributed, but vary according to the deflection at any point, an intermediate value for $\sum r$ may be taken. It will be assumed that

$$\sum r = \frac{6\,p'y}{l} \qquad (2)$$

The moment produced at A-B by each elementary column is equal to the product of $\sum r$ and the distance from the column in question

to A-B. The total moment produced at A-B is equal to the summation of these products across the breadth of the flange. This is the same as the product of the total thrust and the distance from A-B at which it acts. From the assumption that the curve of the elementary cantilever is a parabola it follows that the average value of y is $\frac{1}{3}$ y'. Hence, substituting in (2) the value of $\frac{1}{3}$ y' for y, and the total load P' for p', we determine the total thrust T, tending to cause a moment at A-B to be

$$T = \frac{6\, P' \,\frac{1}{3}\, y'}{l}$$

This thrust is made up of a series of thrusts which are proportional to the values of y, and hence it acts at the center of gravity of the area X O Y. From the properties of the parabola it follows that this center of gravity is distant $\frac{3}{4}$ b from A-B. The bending moment at A-B is therefore

$$M_{A-B} = \frac{6\, P' \,\frac{1}{3}\, y'}{l} \times \frac{3}{4}\, b$$

$$= \frac{3}{2}\, \frac{P'\, y'\, b}{l} \qquad (3)$$

The resisting moment along A-B will now be found in terms of the various dimensions.

The resisting moment of any elementary cantilever beam as a-b is equal to

$$M_r = \frac{2\, y'\, E\, I}{b^2}$$

At any point along the flange edge, the curve being a parabola, we have

$$y = \left(1 - \frac{4\ z^2}{l^2}\right) y'$$

Hence the resisting moment of each cantilever beam is equal to

$$M'_r = y' \left(1 - \frac{4\ z^2}{l^2}\right) \frac{2\ EI}{b^2}$$

Substituting for I its value $\frac{1}{12} t^3 dz$ we have as the total resisting moment in the length l

$$\sum M'_r = \frac{Et^3}{6b^2}\ 2\int_0^{\frac{l}{2}} y' \left(1 - \frac{4z^2}{l^2}\right) dz$$

$$= \frac{Et^3\ ly'}{18\ b^2} \qquad (4)$$

Equating the bending moment (3) and the resisting moment (4) we have

$$\frac{3}{2}\ \frac{P'y'b}{l} = \frac{Et^3 ly'}{18\ b^2}$$

Substituting for P' its value in (1) we get

$$\left(\frac{3}{2}\ ptb - \frac{\pi^2 Et^3 b}{12\ l^2}\right) \frac{by'}{l} = \frac{Et^3 ly'}{18\ b^2}$$

Solving this for p gives

$$p = \frac{Et^2 l^2}{27\ b^4} + \frac{\pi^2 Et^2}{12\ l^2}$$

It will be noted that p depends upon two functions, one of which, the resistance due to cantilever action, varies directly as l^2, and the other of which, the resistance due to column action, varies inversely as l^2. Now l is not necessarily the total length of the flange, but only the length over which flexure in one direction extends. This length may be anything less than the total length of the flange, and so that length giving the least value of p should be taken. Differentiating the above expression, equating to o and solving for l we get l = 2.17 b, say 2.2 b which by trial is found to give a minimum. Substituting this value for l and reducing we get as the final expression for the strength of an outstanding flange,

$$p = 0.35 \frac{E\ t^2}{b^2}$$

This expression is similar to Euler's formula for columns in that it gives the elastic resistance to buckling only, and takes no account of the compressive strength of the material. Since the flange need only be strong enough to take the same unit load as the column as a whole is designed to take, the ultimate compressive strength need not be considered. For any working unit load, the formula fixes the maximum ratio of width of flange to thickness. Thus for a unit load of 15,000 pounds per square inch, we have

$$15{,}000 = 0.35 \frac{30{,}000{,}000\ t^2}{b^2}$$

whence

$$\frac{t}{b} = .038, \quad \text{or} \quad b = 26\ t.$$

There have been few, if any, experiments performed upon columns of ordinary section with the view of determining the strength of flanges. In many tests, however, failure has occurred through the

buckling of flanges, and these tests serve as a guide to the limiting proportions of such parts.

In tests at the Watertown arsenal on rolled H-sections several specimens failed by buckling of the flanges. The detailed dimensions of these sections are not reported but the flanges were about 3 inches wide and 0.39 inches thick. These specimens failed at a load of about 45,000 pounds per square inch. According to the above formula the flanges should have had a strength against buckling greater than this, but as the elastic limit was exceeded the formula could not be regarded as applicable to conditions of failure. There were several built I-columns which also failed by buckling of the flanges. These columns had flanges 4 inches wide and 3/8 inches thick. Failure occurred at about 36,000 pounds per square inch, which was slightly above the elastic limit of the material. According to the formula derived above these flanges had a strength against buckling of

$$0.35\ \frac{30,000,000}{16}\left(.141\right) = 92,500 \text{ pounds per}$$

square inch, indicating that failure would not take place below the elastic limit.

In a series of tests made by Professor Marburg on Bethlehem girders with wide flanges several specimens failed by buckling of the compression flange. These girders had flanges 1 inch thick next the web and 10.5 inches wide. The modulus of rupture as computed from the load at failure was 54,000 pounds per square inch. According to the formula the flanges of these girders were capable of taking a load of

$$0.35\ \frac{30,000,000 \times 1}{25} = 420,000 \text{ pounds per square}$$

inch as far as elastic deflection was concerned.

In all the above cases failure occurred after the elastic limit was reached. According to the above formula the flanges were strong enough to insure against buckling under loads below the elastic limit, and this proved to be the case. The fact that ultimate failure took place by wrinkling of the flanges should not be necessarily regarded as a proof that they were excessively wide.

The most extensive investigation of secondary flexure or wrinkling in compression members was made by Professor Lilly of Trinity College, Dublin. His experiments, however, were made upon circular tubes of small section, and the results are not believed to be directly applicable to columns of usual section. Professor Lilly's tests showed conclusively that for the type of compression member the best results were obtained with a certain ratio of radius to thickness, and that an increase of radius in order to secure a greater section modulus was of no advantage when the thickness was reduced below a certain point. Lilly developed an empirical formula to take account of the wrinkling action. He found that the load producing a wrinkling failure depended, for a given section area, on the ratio of radius of gyration to thickness. From his experiments he concluded that the ultimate unit load was equal to

$$\frac{F}{1 + k\left(\frac{r}{t}\right)^2}$$

where F is the compressive strength of the material, r the radius of gyration of the section, t the thickness of the side, and k an empirical constant, equal to 60 for mild steel. Substituting the above value for the f in Rankine's formula, Lilly derived a formula in which both general and secondary flexure were provided for.

The results obtained from these tests show that for dimensions causing wrinkling failure the effect of a variation in length upon the ultimate load is slight. This would be expected, as secondary flexure takes place over short lengths.

A full account of Professor Lilly's experiments may be found in the Proceedings of the Institute of Mechanical Engineers for June, 1905.

Art. 6. Stresses in Auxiliary Parts.

The question as to the nature and intensity of the stresses in the lattice bars, batten plates and other secondary parts of a column is one of great importance, but at the same time one which hardly admits of a definite answer. These members serve two purposes. First, they furnish lateral support for the main parts of the column. Second, they resist longitudinal and transverse shear in the column. When the load is perfectly central and the column perfectly straight, there is no shear and the stresses in the lattice bars or batten plates, while indeterminate, are in all probability negligible. When the loading is oblique, which is always the case to some extent, the shear may be very considerable and the stresses in the lattice bars correspondingly high. This being the case, it is important that the maximum probable stress should be determined. Various solutions of this problem have been proposed, but most of them depend upon assumptions the correctness of which is open to serious doubt.

One method of calculating the stresses in lattice bars, given in Johnson, Bryan and Turneaure's treatise "The Theory and Practice of Modern Framed Structures" is as follows: The flexural stress indicated by the adopted column formula is computed and a uniform transverse load sufficient to produce the same stress is assumed to

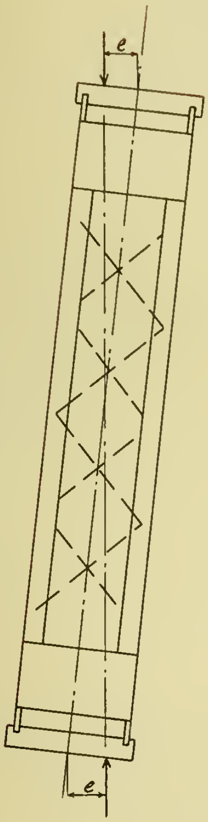
act on the column in the plane of the lattice bars. The shear resulting from this transverse load is assumed to be carried by the lattice bars and the stresses in the latter are calculated accordingly. The fundamental weakness in this method is the assumption of a bending moment equal to that given by the column formula. Although such a formula may give the approximate strength of the column, strain measurements under working loads do not indicate the presence of such stresses as would exist if the column formula were strictly applicable. Furthermore, this method takes no account of possible obliquity of loading, which is really the condition giving the greatest shear.

An example of the unreliability of this method of computing lattice bar stresses is afforded by the Quebec bridge failure. Mr Szlapka used the method, adopting Rankine's formula as given for square ended columns. Instead of assuming an equivalent uniform transverse load, however, he assumed an equivalent concentrated load, which of course gave only half as much shear. He adopted a larger area than that calculated, however, and this increased area proved to be altogether insufficient.

An apparently more rational way of computing the maximum lattice bar stresses than the above method is to assume the greatest obliquity which the column as a whole will stand and to compute the transverse shear and stresses accordingly. In order to obtain the condition of maximum obliquity, an equal and opposite eccentricity is assumed at each end. This method will be illustrated.

A column is assumed under an oblique loading as in Fig. 3.

Fig.3



Let: P = working load on column;

e = eccentricity at each end;

f = stress in steel at the yield point;

A = the area of column section;

I = moment of inertia of the column section; and

c = distance to the extreme fiber of the section from the center of gravity.

Assume that the column is loaded with P, the working load, but with eccentricity sufficient to stress the extreme fiber to the yield point.

Then

$$f = \frac{P}{A} + \frac{Pec}{I}, \quad \text{whence}$$

$$e = \frac{\left(f - \frac{P}{A}\right) I}{Pc}$$

The total obliquity is $\frac{e}{\frac{1}{2} l}$, and the transverse shear being equal to the product of the load and the obliquity we have, denoting by V the shear,

$$V = P \times \frac{e}{\frac{1}{2} l}$$

Substituting the value of e obtained above, we have

$$V = \frac{2\left(f - \frac{P}{A}\right) I}{cl}$$

Knowing the transverse shear the maximum load on the lattice bars can be readily computed. For the conditions here assumed, with equal and opposite eccentricities at the ends, the lattice bars at the middle of the column will be most highly stressed.

It is the practice of some designers to assume a certain angle of obliquity, or in other words, to assume a transverse shear equal to a certain percentage of the direct load. Prichard recommends taking the shear as 1.5 per cent of the load. Some engineers increase the obliquity in order to take account of the curvature, multiplying the angle of obliquity by $\frac{\pi}{2}$ on the grounds that the curve of the column is a sinusoid. If the loading indicated above be assumed it is evident that the greatest transverse shear is at the center, where the bending moment, and hence the curvature, is zero. There would appear, therefore, to be no need of considering the obliquity due to curvature. On the whole, the problem of determining the stresses likely to occur in lattice bars is one which hardly lends itself to a satisfactory theoretical treatment. The conditions which affect the case are too complex and too uncertain to be fully defined. Furthermore, in ordinary design the consideration of rigidity is the determining feature rather than the consideration of strength. Except where the member in question is subject to high secondary stresses rigidity is desirable, and a system of lacing which would be sufficient for any theoretical stresses might be altogether inadequate to render the column stiff and to preserve the integrity of the column section. On this account the adoption of rules for lattice bar design based on experience and tests is on the whole the most satisfactory solution.

Specifications for the size of lattice bars vary. The specifications of the American Railway Engineering Association state:

"The latticing of compression members shall be proportioned to resist the shearing stresses corresponding to the allowance for flexure for uniform load provided in the column formula in paragraph 16 by the term $70 \frac{l}{r}$. The minimum width of lattice bars shall be 2 1/2 inches for 7/8-inch rivets, 2 1/4 inches for 3/4-inch rivets, and 2 inches if 5/8-inch rivets are used. The thickness shall not be less than one-fortieth of the distance between end rivets for single lattice, and one-sixtieth for double lattice. Shapes of equivalent strength may be used".

Cooper's specifications state: "The size and spacing of the lattice bars shall be duly proportioned to the size of the member. They must not be less in width than 2 inches for members 9 inches or less in width, nor 2 1/2 inches for members 12 to 15 inches in width. Single lattice bars shall have a thickness not less than $\frac{1}{40}$ or double lattice bars connected by a rivet at the intersection not less than $\frac{1}{60}$ of the distance between the rivets connecting them to the members."

Ketchum gives practically the same rules.

For cases ordinarily occurring the above specifications give results which are entirely reliable. When however columns must be designed of very unusual size, or to withstand especially severe conditions, recourse must be had to special methods or to individual judgment. What would constitute a sufficient system of lacing for a column taking axial stress only might, for instance, prove altogether inadequate for a column of the same size subject to secondary stress.

It is unfortunate that a greater number of tests have not been made with the object of studying the stresses in lattice bars. The most important tests made so far as those of the University of Illi-

nois, those at the Watertown Arsenal, and those made at Phoenixville on the model chords of the Quebec bridge.

The tests at the University of Illinois were made upon one steel column of especially flimsy design, and upon several wrought-iron bridge ports consisting of channel sections latticed together. Both rolled and built-up channels were used. The make-up of the steel column and of the wrought-iron port upon which most of the lattice bar tests were made are shown on Plate IV.

The columns were tested with pin ends, under both central and oblique loading. Strain measurements were made upon a large number of lattice bars and the stresses and transverse shears calculated therefrom. The stresses were found to be very irregular. The transverse shear calculated from the maximum stresses was found to be very high in some cases, even for nominally central loading. All the results of these tests are given in Bulletin No. 44 of the Engineering Experiment Station of the University of Illinois. Table I, page 38 which is copied from this bulletin, gives the transverse shears calculated from the results of the tests. It will be seen that in some cases they are much higher than the arbitrary values noted under the above discussion.

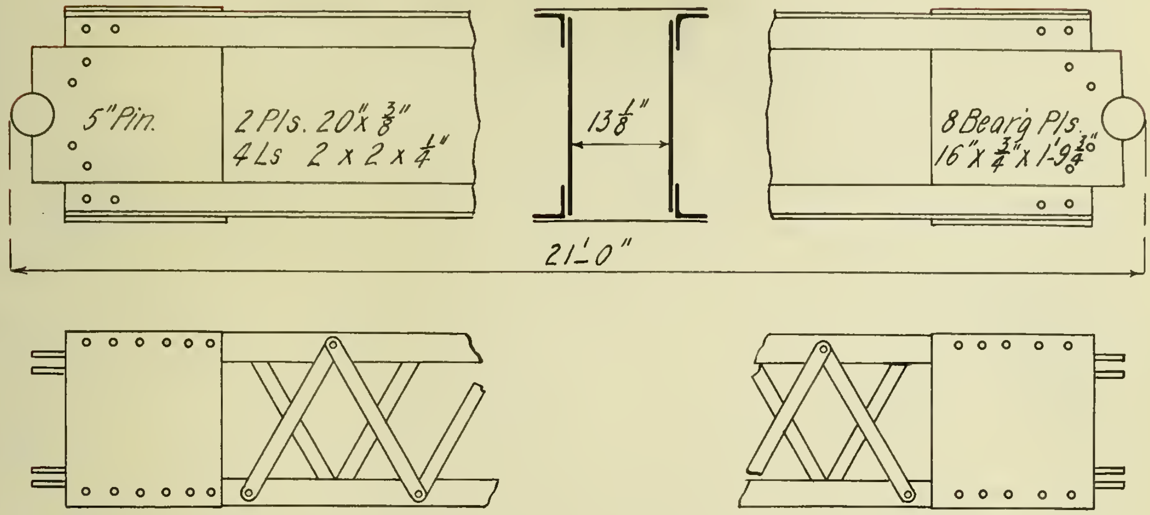
These tests indicated a very uneven distribution of stress across the section of the individual lattice bar. This would be expected, as it is evident that there are other stresses induced than those caused by the transverse shear. The compression of the column causes a change in the angle of inclination of the lattice bars, which can only take place by the bars turning about the rivets at the ends. The grip of the rivets offers sufficient resistance to this turning to cause considerable flexural stresses, and where the bar is rigidly held, as by two rivets at each end, it would seem that

TABLE I.

LATTICE BAR STRESSES. UNIVERSITY OF ILLINOIS TESTS.

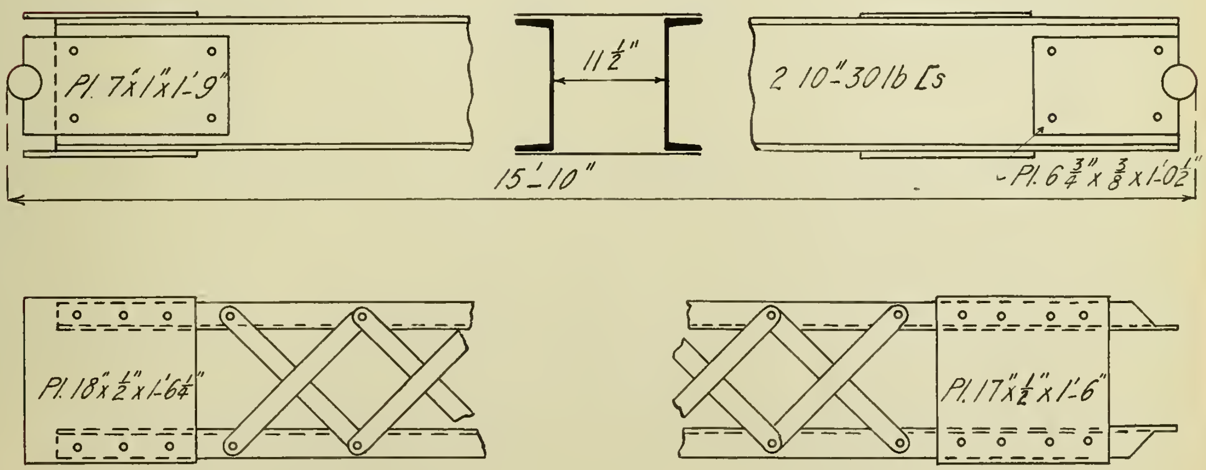
Column	Steel	Steel	Steel	Wrought Iron	Wrought Iron	Wrought Iron
Method of Loading	Central	Central	Oblique Arm 4 in.	Central Central	Oblique Arm 1 in.	Oblique Arm 2 in.
Total Load in Pounds	187,600	187,600	187,600	176,400	176,400	176,400
Maximum Total Stress in Lattice Bar	2,000	1,600	2,700	3,700	4,100	4,100
Corresponding Transverse Shear in Column	3,700	2,900	4,800	5,200	5,800	5,800
Shear Due to Known Eccentricity	0	0	3,000	0	1,000	2,000
Shear Due to Nominal Central Load	3,700	2,900	1,800	5,200	4,800	3,800
Ratio of Shear to Compression Load	.020	.016	.009	.029	.027	.021
Next Highest Observed Values of Ratio of Shear to Compressive Load	.009 .008 .008 .007	.010 .010 .009 .009	.009 .007 .006 .006	.024 .023 .010 .008	.019 .018 .018 .018	.016 .014 .014 .011

UNIVERSITY OF ILLINOIS TEST COLUMNS



Lattice bars 1" x 1/4" and 1 1/4" x 7/16". Slope 63°30'. 1/2" bolts or rivets.

Steel Column



Lattice bars 2 1/2" x 3/8". Slope 45°

Wrought-iron Column

that these flexural stresses might become very important.

The tests made by James E. Howard at the Watertown Arsenal and at Phoenixville were not very exhaustive as regards the investigation of lattice bar stresses, strain measurements being taken on only a few bars. The tests were made on the columns referred to in Art. 4 and shown on Plate III. In no case were the stresses found to be of any considerable amount. These columns were much more compact than those tested at the University of Illinois, and it would therefore be expected that the stresses in the lattice bars should have been less. The fact that so few measurements were taken, however, renders the results of doubtful value in this connection.

Several tests were made at Phoenixville in 1907 on members which were similar in form and proportions to the lower chords of the Quebec bridge. The first test was made upon a chord exactly like the actual bridge member except that all linear dimensions were one-third as great. The end pins used in testing were 12 inches in diameter, which was the size of those used in the bridge. The make-up of this test chord is shown on Plate V.

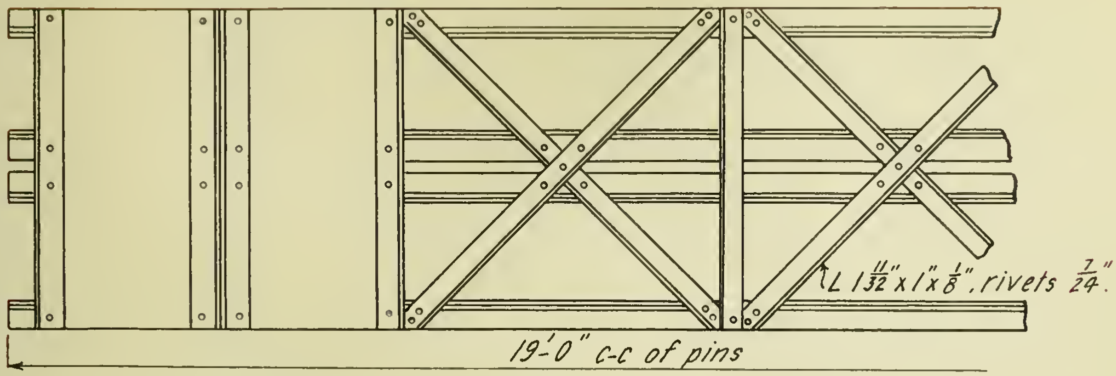
Under a progressively applied load the specimen failed at an average stress of 22,110 pounds per square inch by shearing of the lattice bar rivets near the center of the column. The nature of the failure, which was very sudden, indicated that the main parts of the column had not yet been stressed to the elastic limit. The transverse shear in the member at failure, computed on the basis of the strength of the rivets, was 0.77 per cent of the axial load.

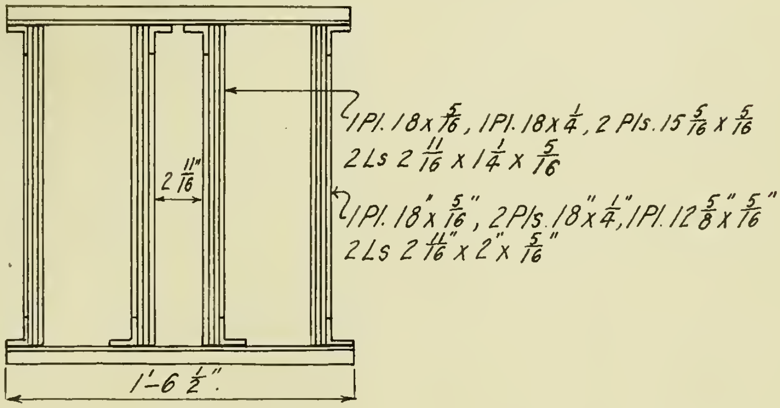
A second test chord was made with lattice bars 50 per cent larger than those of the first and riveted twice as strongly. This chord had only two webs, as shown on Plate V. This specimen failed under a load of 37,000 pounds per square inch by buckling of the webs,

MODEL CHORDS OF QUEBEC BRIDGE

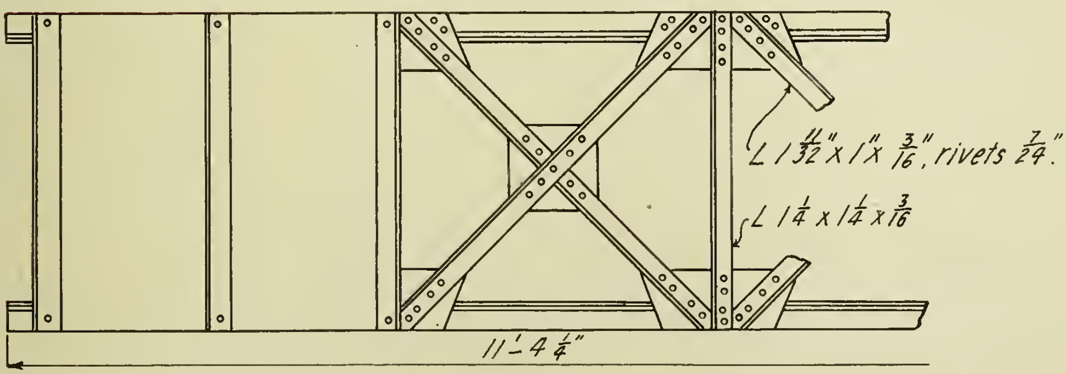
PLATE V

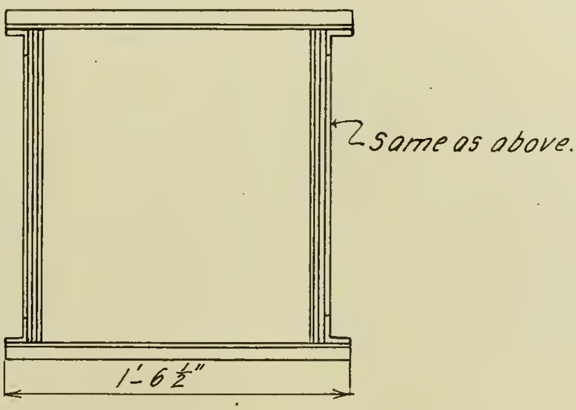
No. 1.





No. 2.





indicating that the lattice bars were sufficient to develop the full strength of the member.

IV. EXPERIMENTAL INVESTIGATION OF THE EFFECT OF RIVETING.

Art. 7. Object of Tests.

It is commonly assumed in the design of compression members that the gross area is effective in taking stress, the rivets being considered as taking the place of the metal punched out of the holes. Whether this assumption is fully justified or not is a question. It is reasonable to suppose that the rivet, in cooling, should contract and leave the hole only partially filled. Tests on riveted joints show that this is really the case. Low stresses are resisted entirely by the friction of the rivet heads against the sides of the connected members and considerable slipping may take place before the rivets become effective in direct shear. This being the case, it would appear that in a compression member some deformation would take place before the sides of the rivet holes came to an actual bearing on the rivets. This deformation could not be considered as resisted by the full section of the member.

If it is true that up to a certain load the rivets are not effective in taking direct compression it would be expected that conditions of stress in a riveted member would be similar to those obtaining in a member similarly punched but having the holes unfilled. Some resistance would be expected from the grip of the rivet heads; but in any event the distribution of stress would not be as regular as in a perfectly plain, solid member.

It was for the purpose of determining what effect the rivets did have on the behavior of compression members under load that these tests were made. It was purposed to discover not only the effect

upon the ultimate strength, but also upon the distribution of stress across the section and on the elasticity or resiliency of the specimen.

Art. 8. Tests.

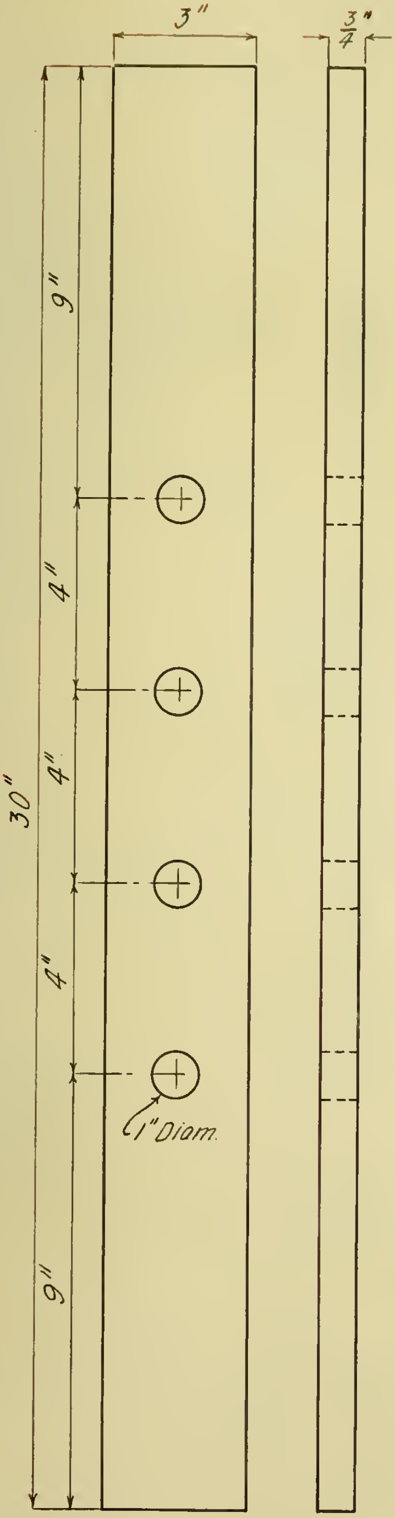
The test pieces were of ordinary structural steel. They were made by the Chicago Bridge and Iron Works, and it is believed that the riveting is representative of ordinary commercial work.

The form of test piece adopted was the plain rectangular bar, as shown on Plate VI. It was desired that the section be such as to permit of the greatest possible area being taken up by the rivets. It was also necessary that the member be sufficiently heavy not to be seriously injured during the process of punching and riveting, and yet not so large as to be beyond the capacity of the 100,000-pound used in making the tests. For these reasons the form adopted seemed best.

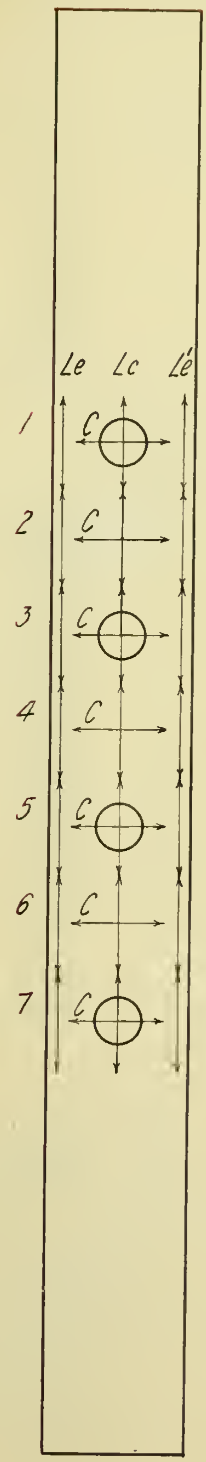
On account of the slenderness of the specimens it was necessary to test them with fixed ends in order to prevent buckling under low stresses. This was accomplished by arranging the machine as shown on Plate VII. The upper cross head was removed and placed upside down on the base of the machine. The specimen could then be clamped by wedge grips at each end and a condition closely approximating that of a true fixed-ended column obtained.

Nine specimens were tested. Three of these were plain bars, three had holes punched as shown on Plate VI, and three had rivets in the holes. It was believed that by testing the three different kinds of specimens in the same manner the effect of riveting could be determined fairly well.

TEST PIECES PLATE VI



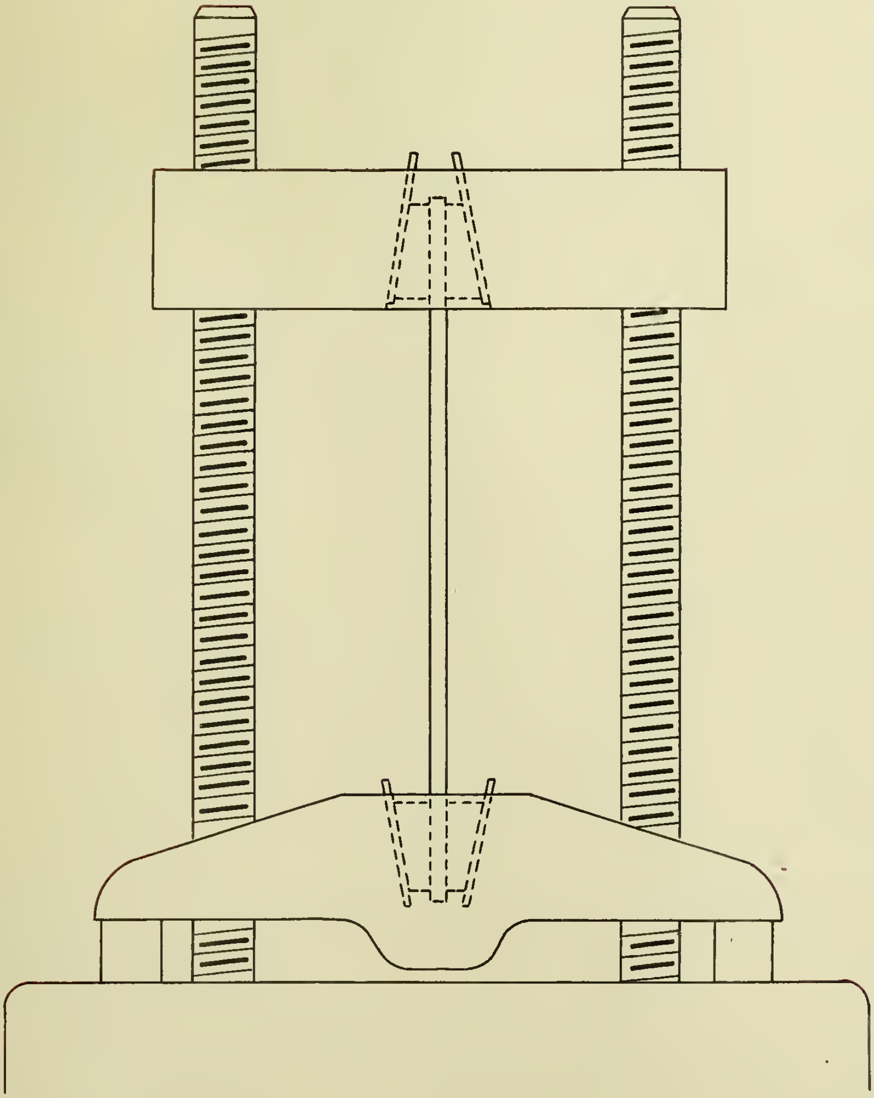
Form of Test Piece



Location of 2-inch gauged lengths.

PLATE VIII

ARRANGEMENT FOR TESTS



Strain measurements were made over two inch gauged lengths, both longitudinally and transversely of the member. The longitudinal or axial measurements were taken along the edges and along the center of each face, and the transverse measurements at every two inches across and between the rivets. The lines along which measurements were taken, together with the nomenclature used to distinguish them, are shown on Plate VI . In making the measurements a Berry extensometer was used. This instrument read directly to 0.0002 of an inch and by estimation to 0.00002 of an inch.

The general routine of testing was as follows: The specimen to be tested was clamped in the machine with a free length of 20 inches between grips. An initial load of 1,000 pounds per square inch of gross area was applied and an initial reading taken on each gauge length. The specimen was then loaded up to 10,000 pounds per square inch and then by increments of 5,000 pounds per square inch each up to failure, readings being taken after the application of each load. At two, and sometimes three stages, the load was taken off and the permanent set determined. In all cases some time was allowed to elapse between the application of any load and the taking of the readings, in order that the specimen should have time to fully adjust itself to the load.

Some difficulty was experienced in maintaining the higher loads, the specimen yielding slightly under continued load even at average unit stresses considerably below the yield point. In most cases the load fell off only three or four hundred pounds while the readings were being taken, but in one or two instances the decrease was as much as a thousand pounds.

There was not sufficient time available to permit of the

carrying out of all the tests according to the plan given above. Those made may be divided into two sets. Set I consists of tests made on two specimens, one containing rivets and one containing open holes. On these the full series of strain measurements was made. Set II consists of tests on three specimens, one containing open holes, one containing rivets and one plain. In the tests strain measurements were made on the gauge lengths Le, Lc, and Le 3, 4 and 5, and on C 3, 4 and 5 on both faces. All three specimens were tested in as nearly the same way as possible and with great care. It is believed that the results of these tests are more reliable than those of Set I. The remaining four specimens were simply tested to failure in order to determine the ultimate strength of each.

Art. 9. Discussion of Data and Results.

On Plates VIII to IX inclusive, pages 55 to 56 are shown the load deformation curves for the specimen of Set I containing open holes. The curves on Plate VIII are from strain measurements taken on the face concave after failure, those on Plate IX from measurements taken on the convex face.

It is apparent that the stress was very unevenly distributed throughout the member. Along Le on the concave face the deformation seems to have been fairly uniform, although slightly greater next the holes. Along Lc the deformation measured across the holes was very much greater than that between the holes, and also greater than that at any other point. Along Le' the deformation was considerably greater next the holes. On the convex face of the specimen the measurements indicated the same conditions along Lc and were found on the concave face. Along Le and Le', especially the former, the deformation was greater between the holes than next the holes. This

is exactly the opposite of what was the case on the other face. The reason for this apparently is as follows: The process of punching left a bowl shaped depression on one face of the specimen and a corresponding bulge on the other face. This dishing extended clear across the member and hence, at the holes, the measurements were taken over the depression on the one face and over the bulge on the other. It is evident that higher strains would occur on the side of the depression, and as this side was always concave after failure the apparently contradictory results obtained on the first specimen would also be expected from the other tests. This will be seen to be the case.

In order to determine accurately the conditions of stress adjacent to the holes a more refined method than that used in these tests would be necessary. The crowding together of the lines of force at this point, together with flexual stresses in the parts to either side of the hole renders the stresses in the entire neighborhood very complex.

As before remarked all the strain measurements along the center lines Lc of the member containing holes show a uniform excess in deformation across the hole. On pages 57 & 58 are shown the results obtained from the specimen containing rivets and it will be noted that this condition prevails, though in a somewhat less degree. On page 59 are shown the curves for transverse deformation for both specimens. It will be observed that the lateral expansion across the holes was very much greater than between the holes, and in the riveted specimen the inequality is only slightly less marked.

On Plates XII to XVI inclusive, pages 59 to 63 , are shown the results obtained from the tests of Set II. The load deformation for the three specimens are plotted side by side in order to afford

a means of direct comparison.

The variation in behavior as indicated by these curves is very striking. In general the curves for the member containing rivets are intermediate between those for the specimen containing holes and those for the plain specimen. The uniformity of the curves for the plain specimen is in striking contrast to the eccentricity of the others. The effect of column action was more marked in the specimens which had been punched, the strains on the concave face being much greater than those on the convex face, and this difference becoming apparent at a low unit load. The reason for this is undoubtedly the distortion due to punching.

On Plate XV is shown the curves for lateral deformation of the specimens of Set II. They are similar to the curves of Set I, except that the results on the plain specimen are included. The curves on Plate XVI show the permanent sets observed on the specimens of Set II after the application of different loads. Here the difference between the riveted and the plain specimens was not very marked up to unit loads of 15,000 or 20,000 pounds per square inch; but beyond that point measurements over rivets showed a rapid increase of set in the riveted specimen. The specimen with open holes showed excessive set, as would be expected from the high deformations observed.

The results of these tests are qualitative rather than quantitative. They show that in the members containing unfilled holes practically all the load was carried by the metal between the edges of the holes and the edges of the specimen, the material between holes taking very little stress. The lateral deformation was abnormally great, especially across the holes. Considerable set took place at low unit loads and the elastic limit and ultimate strength

if based on the gross area, was low.

These curves show that the same things are true, though in less degree, of the members which contained rivets. The fact that the deformation, both axial and transverse measured across rivets was considerably greater than the corresponding deformation measured between or beside the rivets shows conclusively that the rivets do not take the place of the metal removed in punching. There was no indication that the rivets became effective in direct compression at any time. It was thought that after a certain unit load was reached the sides of the holes would come to a bearing against the rivets, but there is nothing in the results to indicate that this was really the case. It does appear, however, that considerable resistance was afforded by the grip of the rivet heads against the sides of the specimen, especially at unit loads up to about 18,000 pounds per square inch. The strength of the riveted specimens was considerably greater than that of the specimens containing unfilled holes.

In the plain specimens the load-deformation curves are for the most part very uniform and regular. They indicate a modulus of elasticity of about 29,000,000 pounds per square inch and a valve of Poisson's ratio of about 0.25 or 0.30.

The values obtained for the ultimate strength of the various specimens are given in Table II.

TABLE II. RESULTS OF TESTS.

Kind of Specimen	No.	Ultimate Strength in Pounds per Square Inch of Gross Area	Average Strength	Ratio of Average Strength to Average Strength of Plain Specimens
With Holes	1	23,750		
	2	22,700		
	3	23,200	23,500	0.790
With Rivets	1	28,600		
	2	27,500		
	3	27,100	27,700	0.925
Plain	1	29,800		
	2	29,700		
	3	29,900	29,800	1.00

It will be observed that the riveted specimens were 92.5 per cent as strong as the plain specimens, while those containing holes were only 79 per cent as strong. The strengthening effect of the rivets is thus very clearly shown.

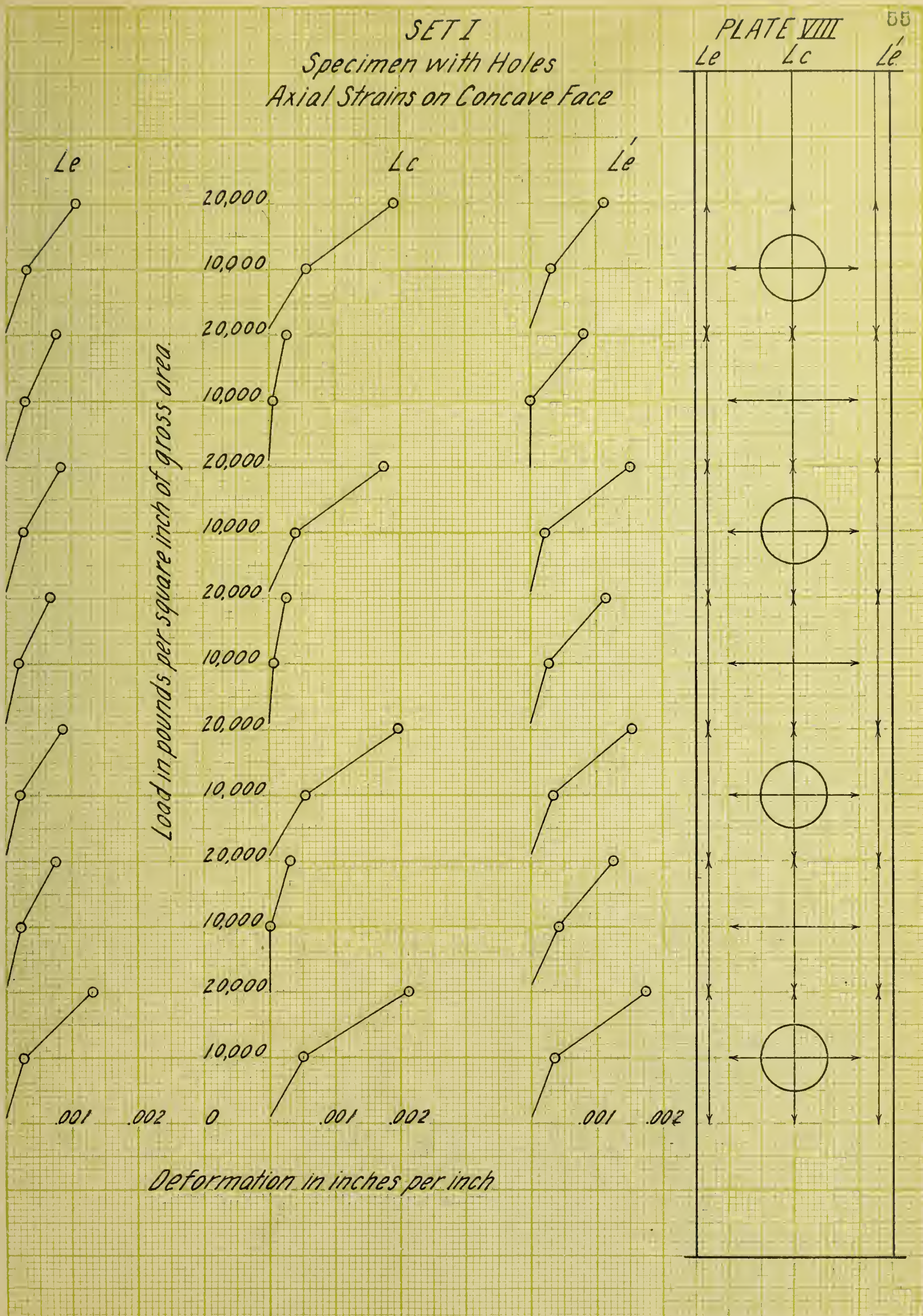
In these specimens the rivets occupied one-third of the section. The pieces containing rivets being 92.5 per cent as strong as the plain pieces, the efficiency of the rivets, based on the ultimate strength, was 77.5 per cent.

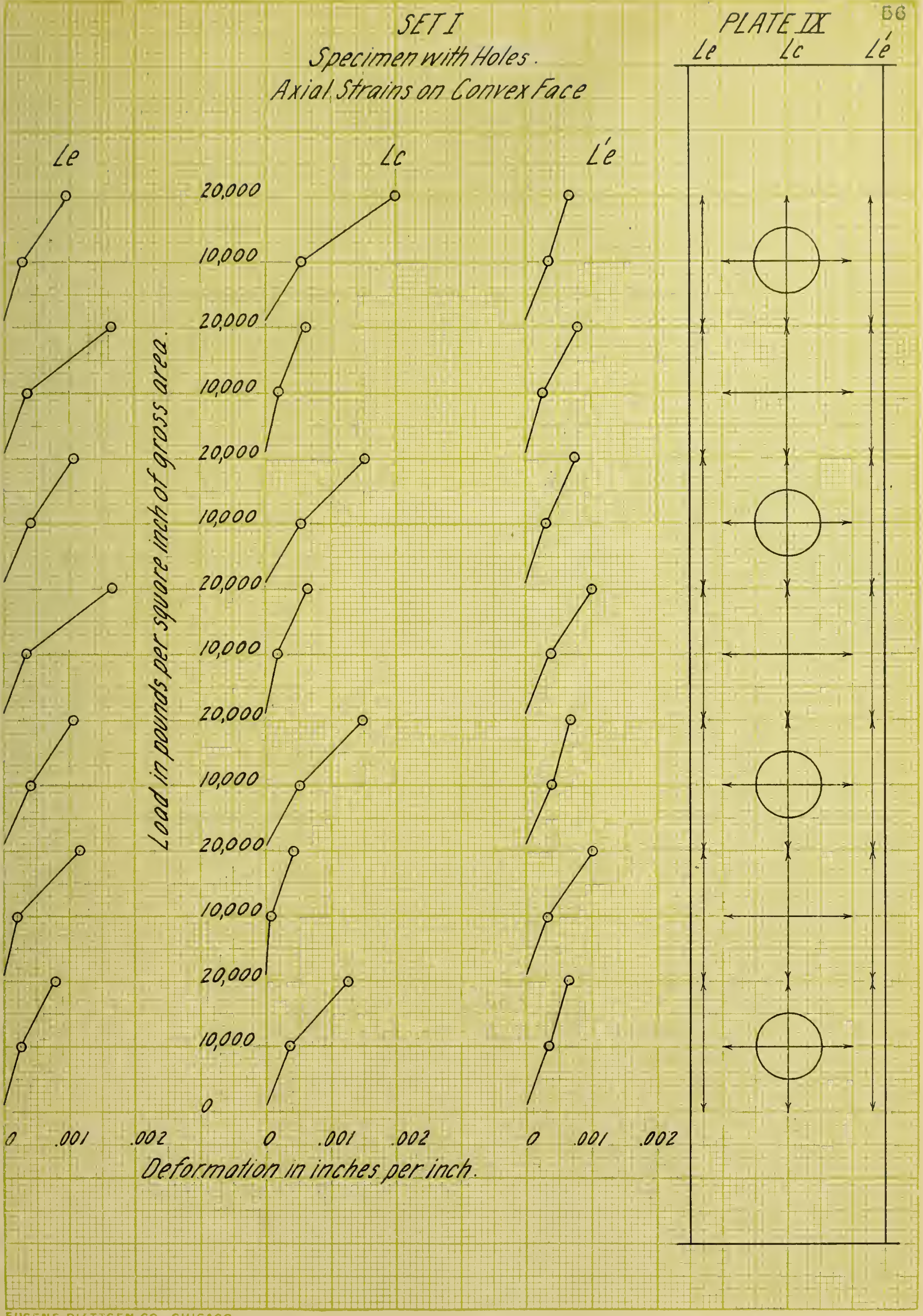
It is probable that much of the difference in strength between the plain and the riveted specimens was due to the distortion of the latter caused by punching. There is another thing which might have further weakened the riveted specimens. It was found that the legs of the extensometer were not long enough to straddle the rivet heads, and so the latter were machined off for about half their height. Undoubtedly rivets are ordinarily in a state of high tension due to cooling. This tension causes flexural and shearing stresses in the rivet heads, and it is possible that the cutting off of part of the latter reduced the area opposed to the shear and flexure sufficiently to relieve the tension in the rivets. This would reduce the resistance afforded by the friction of the rivet heads against the specimen.

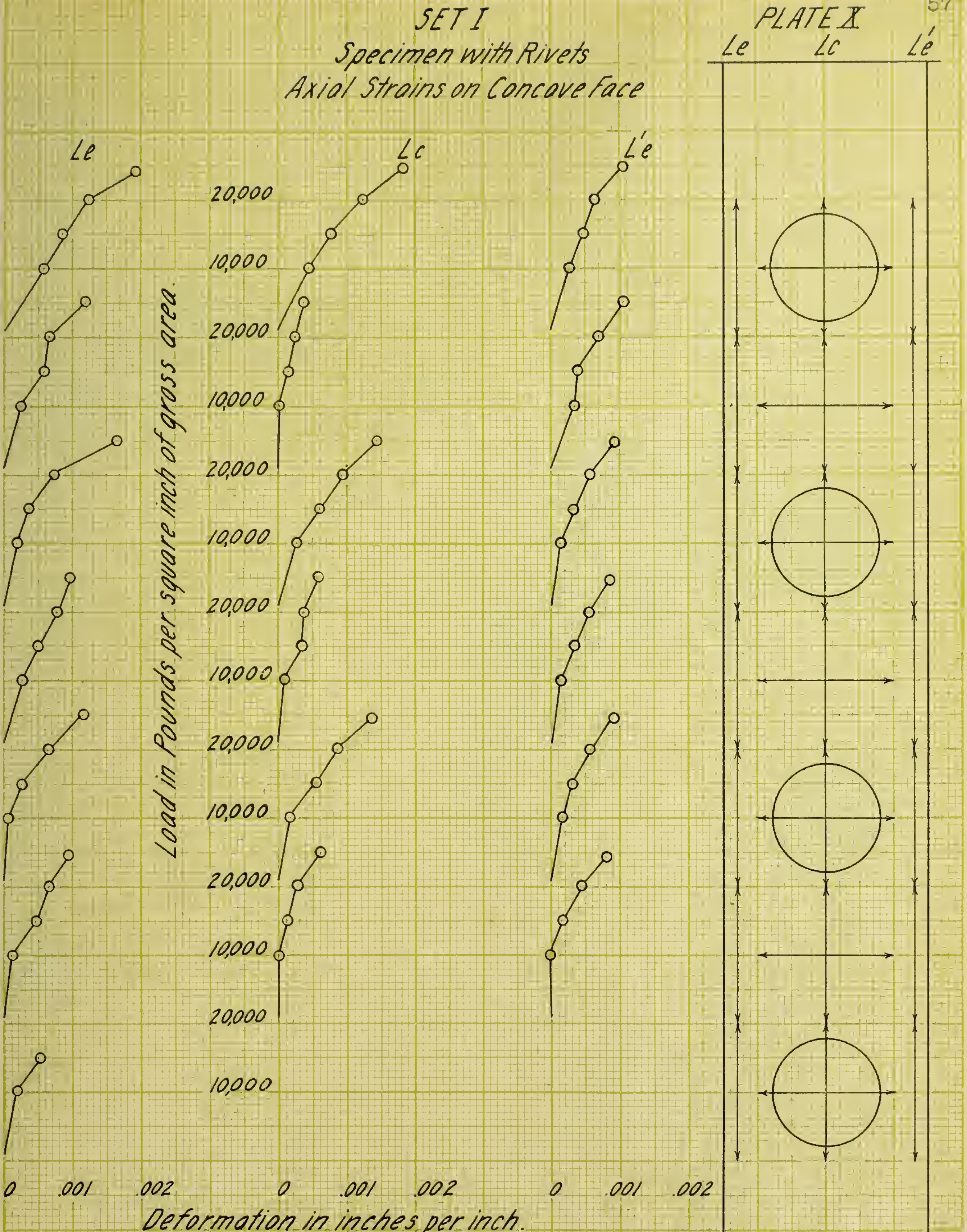
It is evident that the elastic limit was locally exceeded on the riveted specimens at lower loads than on the plain specimens. In the case of the former, the elastic limit at points along Le and L'e was usually reached at a unit load of about 18,000 pounds per square inch or less. In the case of the latter there are no indications that the elastic limit was reached even locally at unit loads of less than 22,500 pounds per square inch. This

difference was probably due largely to the initial stresses and deformation, due to punching, in the riveted specimens.

To fairly determine the real efficiency of rivets in resisting compression, the holes should be drilled and the rivets left entire. At the same time it should be remembered that in actual structures riveting is always attended with the deformation due to punching. If there is a consequent reduction in strength, as would appear from these tests, some allowance should be made in cases where there is a large amount of riveting. Ordinarily the proportion of the section taken up by rivets is much smaller than was the case with the specimens used in these tests. In some members, however, as in the compression flanges of plate girders, a very considerable part of the section is so taken up. Whenever the rivets occupy a certain proportion of the section area, say 20 per cent, allowance might be made by considering the effective area as the net area plus about 75 per cent of the rivet area. Just how much such a provision is required can only be determined by extensive and varied tests.


SET I
Specimen with Holes
Axial Strains on Concave Face
PLATE VIII
Le
Lc
Le'
20,000
10,000
Load in pounds per square inch of gross area.
.001
.002
0
Deformation in inches per inch


SET I
Specimen with Holes.
Axial Strains on Convex Face
PLATE IX
Le
Lc
L'e
20,000
10,000
0
Load in pounds per square inch of gross area.
0
.001
.002
Deformation in inches per inch.


SET I
Specimen with Rivets
Axial Strains on Concave Face
PLATE X
Le
Lc
L'e
Load in Pounds per square inch of gross area.
20,000
10,000
0
.001
.002
Deformation in inches per inch.

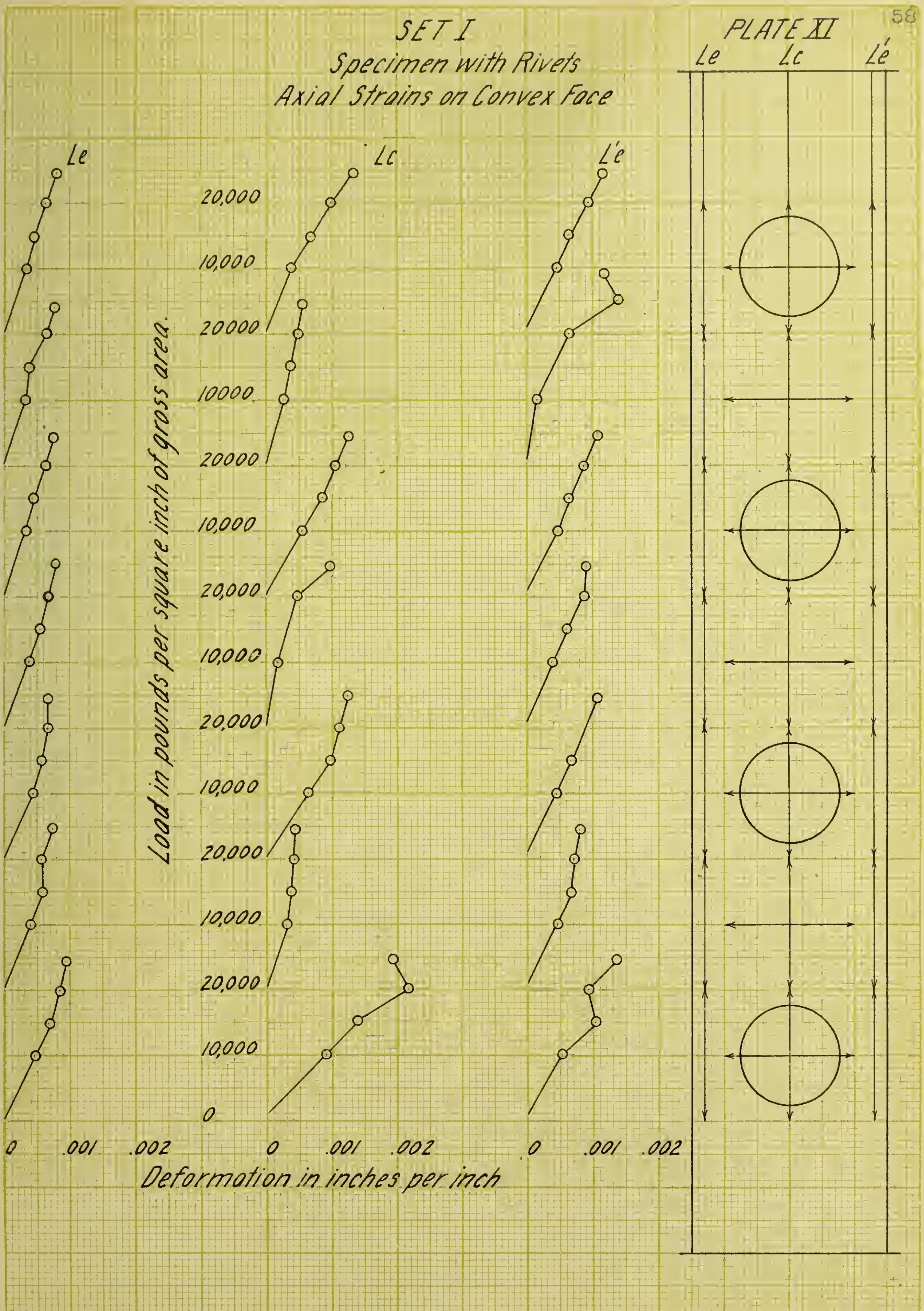
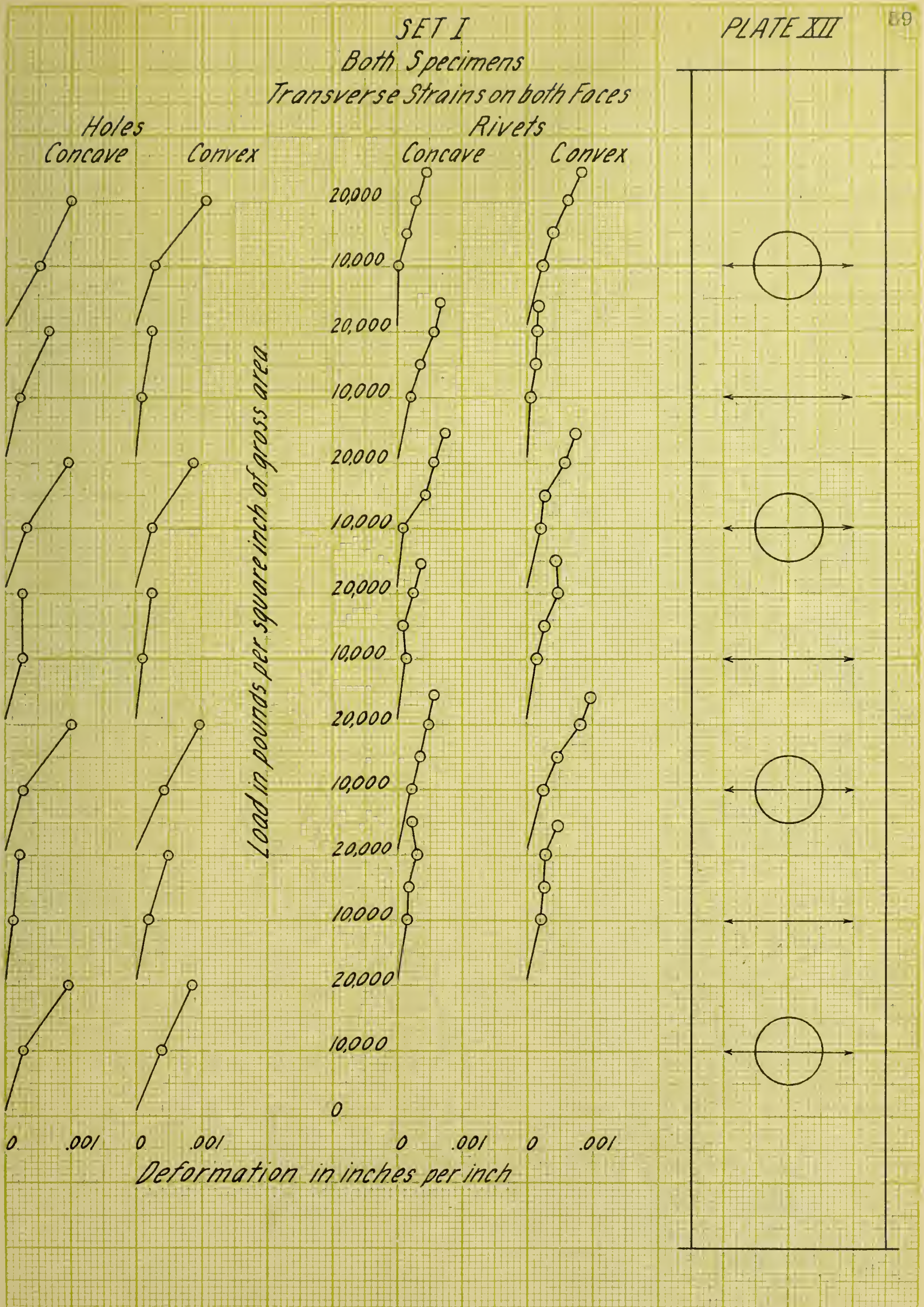
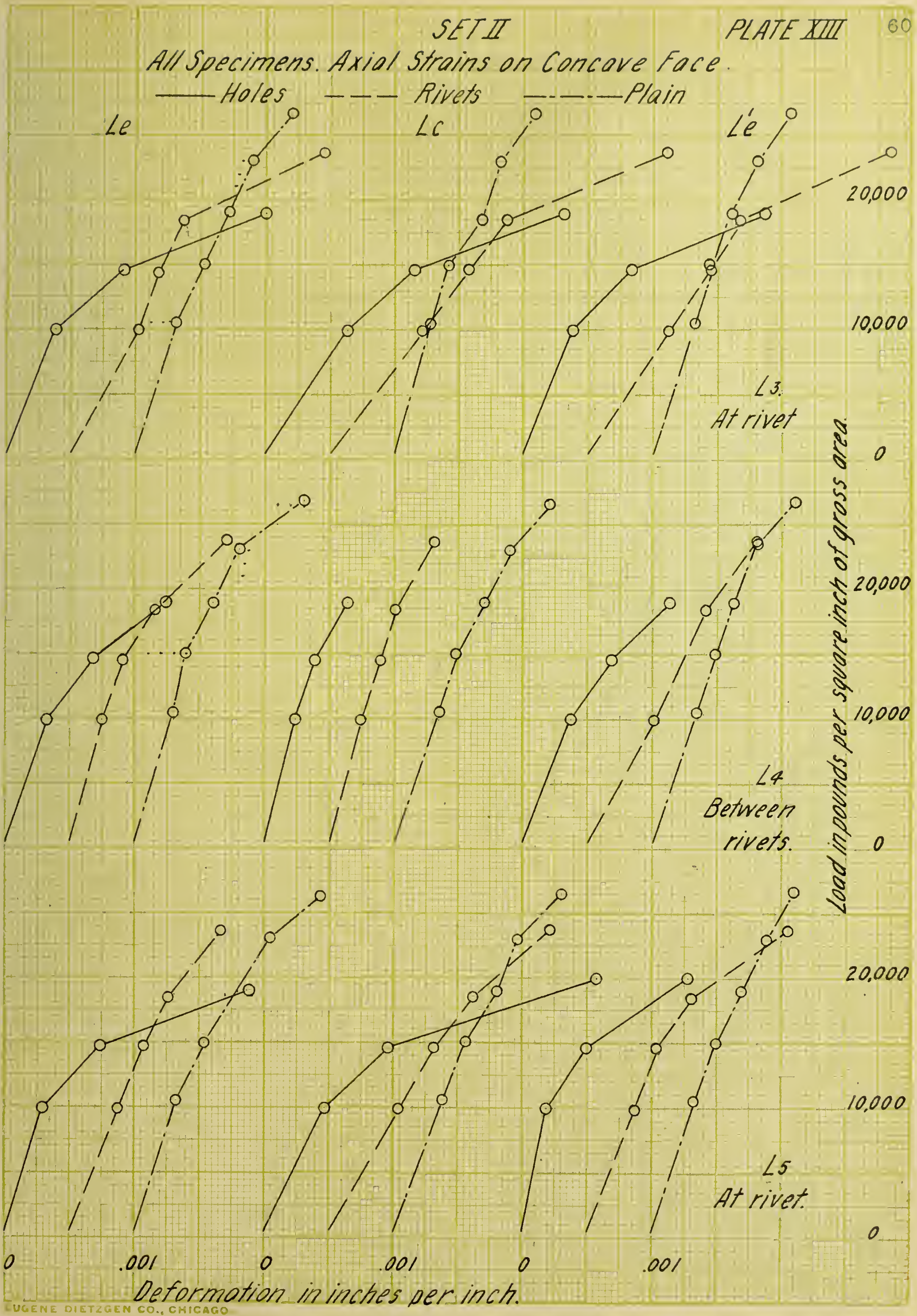
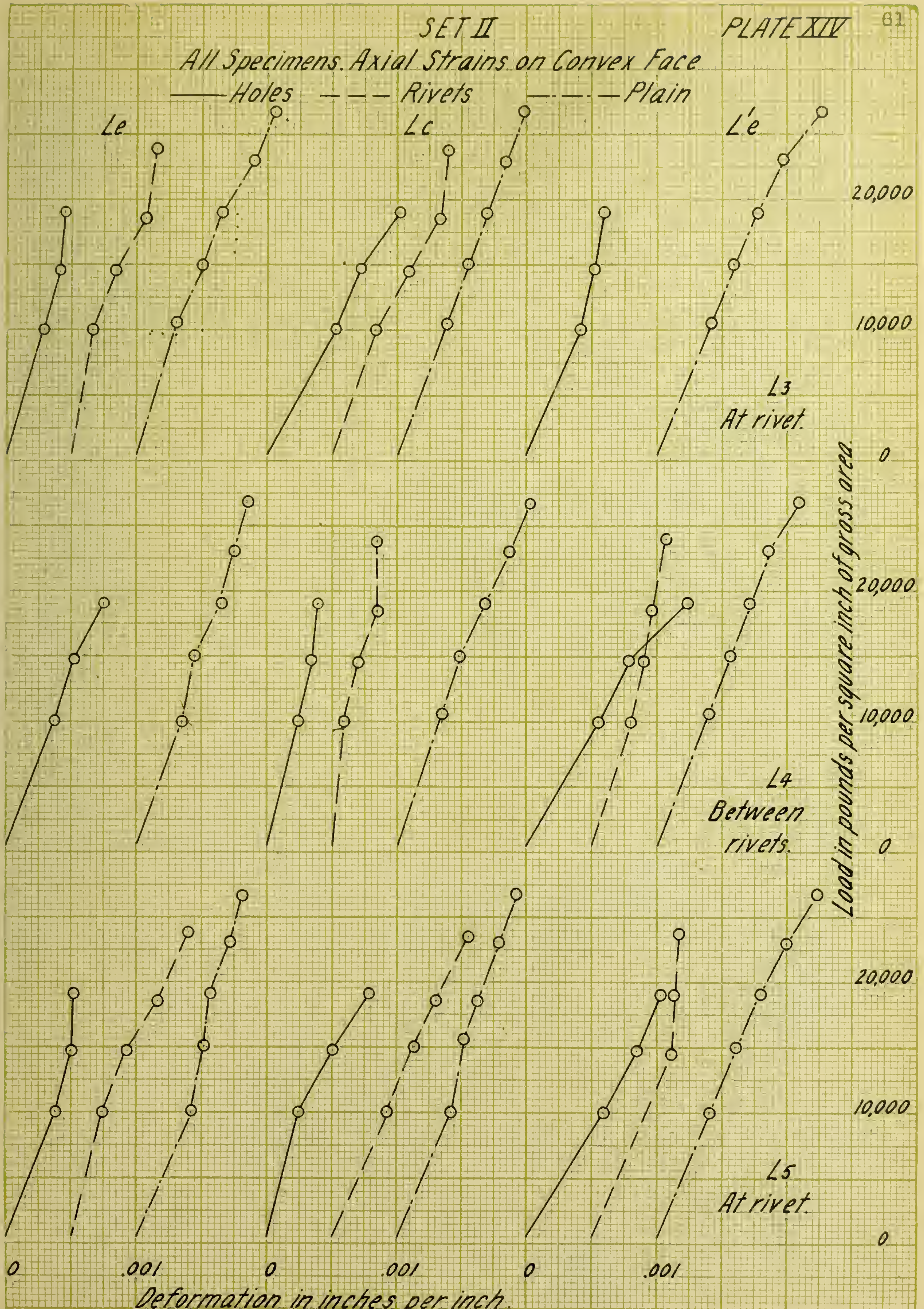
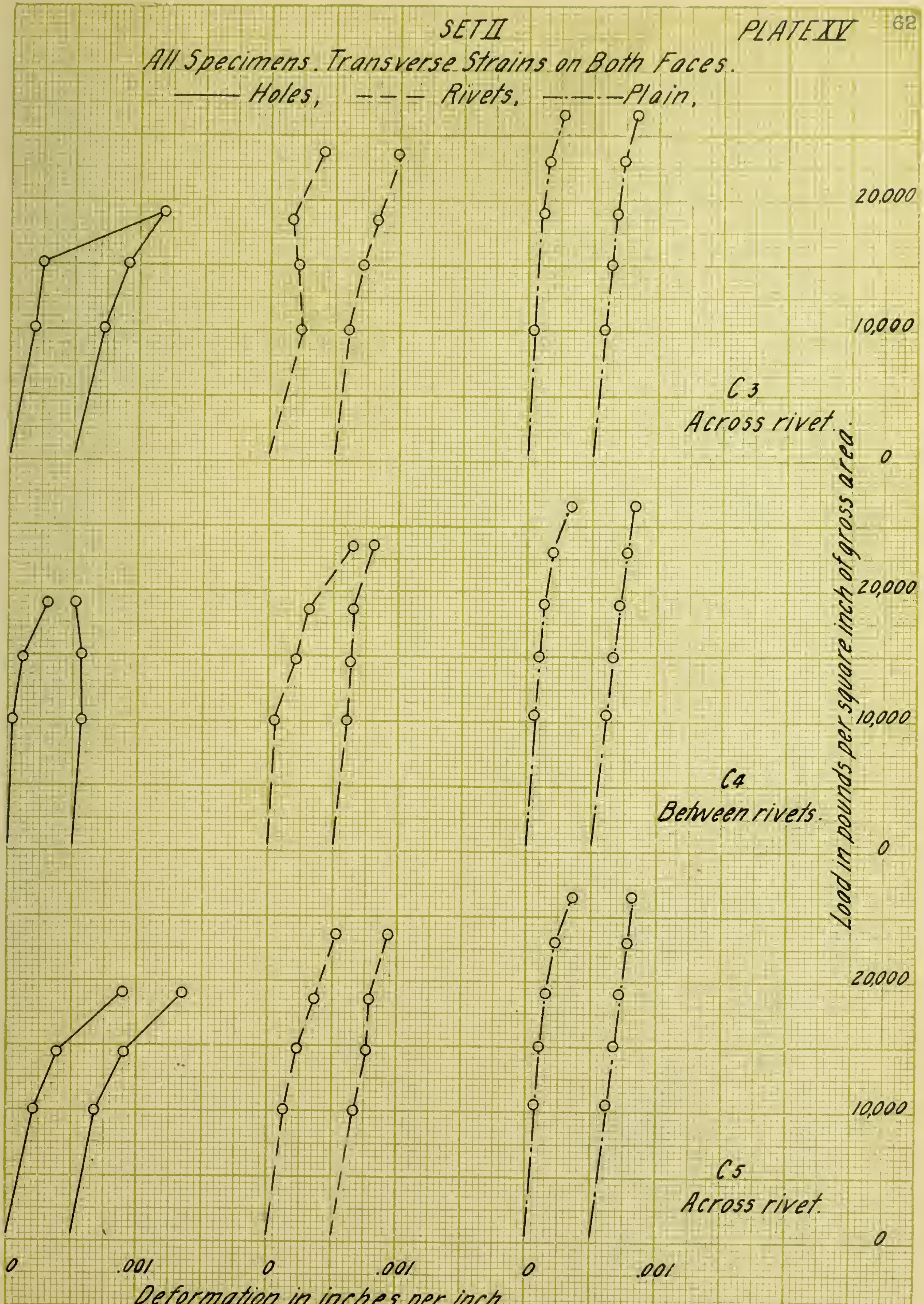


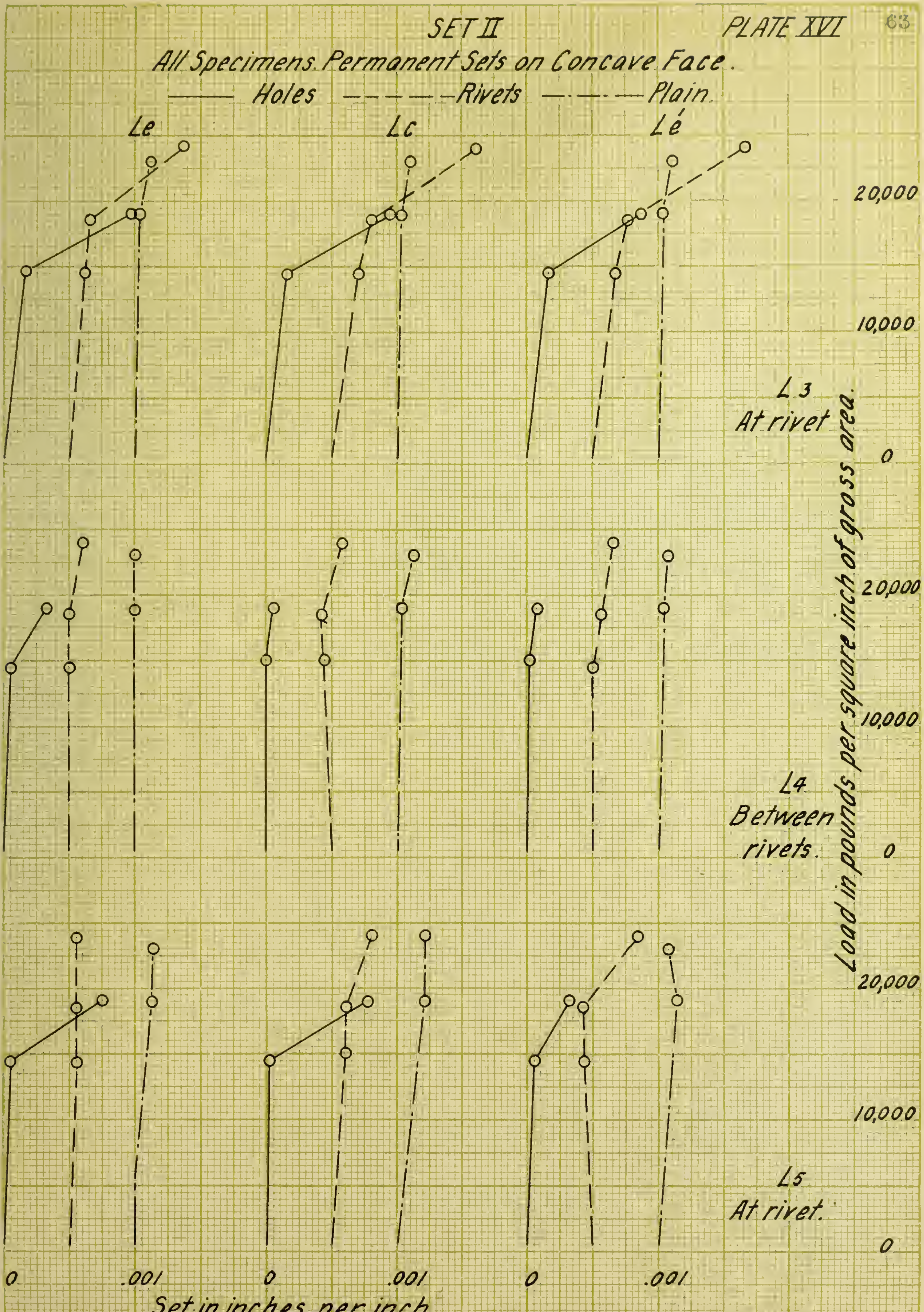
PLATE XI
SET I
Specimen with Rivets
Axial Strains on Convex Face
Le
Lc
L'e
20,000
10,000
20000
10000
20000
10,000
20,000
10,000
20,000
10,000
20,000
10,000
20,000
10,000
0
Load in pounds per square inch of gross area.
0 .001 .002
0 .001 .002
0 .001 .002
Deformation in inches per inch
Le Lc L'e


SET I
PLATE XII
Both Specimens
Transverse Strains on both Faces
Holes
Concave
Convex
Rivets
Concave
Convex
20,000
10,000
20,000
10,000
20,000
10,000
20,000
10,000
20,000
10,000
20,000
10,000
20,000
10,000
0
Load in pounds per square inch of gross area.
0
.001
0
.001
0
.001
0
.001
Deformation in inches per inch


SET II
PLATE XIII
All Specimens. Axial Strains on Concave Face.
Holes
Rivets
Plain
Le
Lc
L'e
20,000
10,000
0
L3.
At rivet
L4
Between
rivets.
L5
At rivet.
Load in pounds per square inch of gross area.
0
.001
Deformation in inches per inch.


SET II
PLATE XIV
All Specimens. Axial Strains on Convex Face
Holes
Rivets
Plain
Le
Lc
L'e
20,000
10,000
L3
At rivet.
0
20,000
10,000
L4
Between
rivets.
0
20,000
10,000
L5
At rivet.
0
Load in pounds per square inch of gross area.
0
.001
0
.001
0
.001
Deformation in inches per inch.


SET II
PLATE XV
All Specimens. Transverse Strains on Both Faces.
Holes, Rivets, Plain.
20,000
10,000
C3
Across rivet.
0
20,000
10,000
C4
Between rivets.
0
20,000
10,000
C5
Across rivet.
0
Load in pounds per square inch of gross area
0
.001
0
.001
0
.001
Deformation in inches per inch


SET II
PLATE XVI
All Specimens. Permanent Sets on Concave Face.
Holes
Rivets
Plain.
Le
Lc
Lé
20,000
10,000
0
L3
At rivet
20,000
10,000
0
L4
Between
rivets.
20,000
10,000
0
L5
At rivet.
Load in pounds per square inch of gross area.
0
.001
0
.001
0
.001
Set in inches per inch.

V. CONCLUSIONS.

From the study of column formulas and column tests discussed in Part II the following conclusions are drawn.

(1) None of the column formulas now in use can be relied upon to give accurate results when applied generally to columns of different forms and dimensions.

(2) The variation of column strength with the value of the ratio $\frac{l}{r}$ does not appear to be either so marked or so consistent as is indicated by the usual formulas. For values of $\frac{l}{r}$ most commonly occurring in practice a horizontal straight line may be considered as defining the strength of columns practically as well as the curves of the various formulas.

(3) Various features, such as form of section, rigidity of lacing, and liability to initial stress and deformation, play a very important part in limiting the strength of a column.

(4) In consideration of these facts, the adoption of a constant unit working load for columns having slenderness ratios within certain limits would seem to be justified. For columns having values of $\frac{l}{r}$ between 40 and 80 a working load of 12,000 pounds per square inch could be adopted. For lower values of $\frac{l}{r}$, the unit working load could be determined by the formula

$$p = 16,000 - 100 \frac{l}{r} .$$

For values of $\frac{l}{r}$ from 80 to 100 the following formula could be used

$$p = 24,000 - 150 \frac{l}{r} .$$

For the limits of $\frac{l}{r} = 0$ and $\frac{l}{r} = 100$ these formulas would give the same values as the formula for working load recommended by the American Railway Engineering Association, which is

$$p = 16,000 - 70 \frac{l}{r} .$$

(5) The results given by the Navier and the Moncrieff-Merriman formulas for stresses due to eccentric loading are fairly consistent. The latter formula is the more rational, gives higher values for the stress, and in the absence of experimental data probably should be used in ordinary cases.

From the study of stresses in columns, discussed in Part III, the following conclusions are drawn.

(1) The distribution of compressive stress across the section of a built-up column is irregular, and varies more or less throughout the length of the member. This irregularity is more marked in columns composed of thin, flimsy parts than in columns which are stocky and compact.

(2) When a column is loaded through a pin, compressive stresses of high intensity are developed immediately in front of the pin. These stresses are higher than would be the case if the pressure were uniformly distributed over an area equal to the thickness of the bearing surface and the diameter of the pin. On this account it would seem that the bearing stress ordinarily specified was too high.

(3) In columns composed in part of relatively thin webs and flanges, the strength of the member may be limited by the resistance of such parts to local flexure or buckling. In the case of an outstanding flange the resistance to elastic buckling may be

expressed by the formula

$$p = 0.35 \frac{E\ t^2}{b^2}$$

where p is the load in pounds per square inch, E the modulus of elasticity, and t and b the thickness and breadth of the flange respectively. As this formula is based upon several assumptions it should, in the absence of any experimental verification, be regarded as tentative.

(4) The theoretical determination of stresses in lattice bars is impracticable. In view of the importance of a rigid connection between the parts of a column the lattice system should be considerably heavier than would be necessary to simply resist the probable shear in the member.

The results of the tests made to determine the effect of riveting appear to justify the following conclusions.

(1) The distribution of stress in members having a considerable reduction in area due to holes is very irregular. The stress on the net area is high and the longitudinal and lateral deformations across holes are extreme.

(2) In a riveted member the rivets do not fill the holes and hence do not serve to take compression directly at working loads. The grip of the rivet heads is sufficient to afford very considerable resistance to deformation, and the riveted member is thus much stronger than a similar member containing open holes.

(3) The distortion of the member caused by punching renders it more liable to failure by buckling. It seems probable that the resistance to local flexure would be especially affected.

(4) In members having a large portion of the section area taken up by rivets the gross area should not be regarded as fully effective in taking compression. According to these tests about 75 per cent of the area occupied by the rivets could be regarded as effective. The proper allowance can only be determined by extensive tests.